Nuclear Reactions II: Applications to Nuclear Structure

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Oak Ridge, June 28 2019

FRIB

Outline

1. Why do reactions? Probing nuclear structure
   - Elastic scattering
   - Inelastic scattering
   - One-particle transfer

2. An advanced example: 2-neutron transfer and pairing
   - Pairing correlations and successive transfer
   - Reaction and structure models
   - A quantitative measure of pairing correlations

3. Moving forward: integrating structure and reactions
   - (d, p) reactions: a unified approach
   - Choosing the potential: examples of calculations

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Elastic scattering

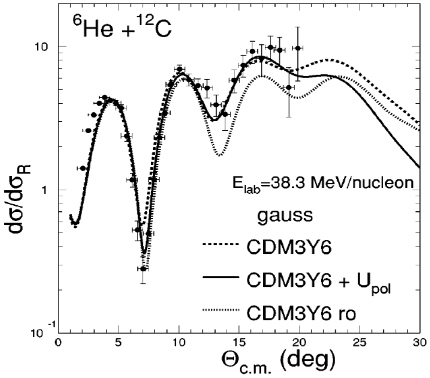
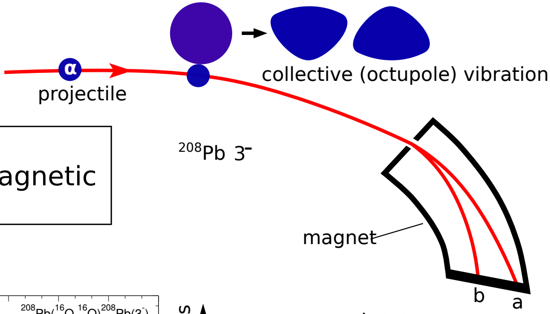


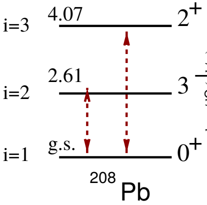
FIG. 10. Elastic scattering for ^{6}He+ ^{12}C at 38.3 MeV/nucleon in comparison with the OM results given by the real folded potential (obtained with the CDM3Y6 interaction and the Gaussian *ga* density for ^{6}He). The dashed curve is obtained with the unrenormalized folded potential only. The solid curve is obtained by adding a complex surface polarization potential to the real folded potential. Its parameters, and those of the imaginary part, are explained in the text. The dotted line is obtained by folding the CDM3Y6 interaction with the compact Gaussian density *ro*.

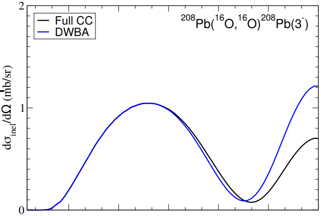
[Lapoux et al, PRC 66 (02) 034608]

traditionally used to extract optical potentials, rms radii, density distributions.

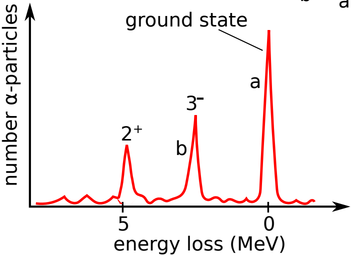


used to extract nuclear deformations, electromagnetic transitions...

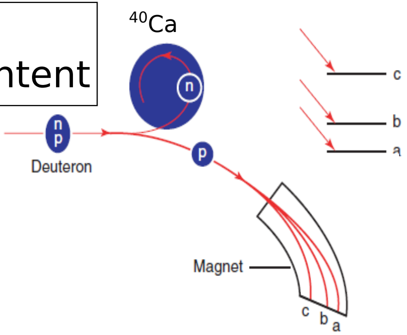




A. Moro

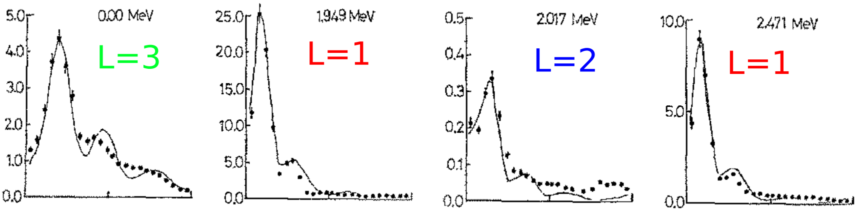


populates states with strong single-particle content



shape: angular momentum.

magnitude: spectroscopic factor (single-particle strength).



Brown *et al.* Nucl. Phys. A **225** (1974) 267

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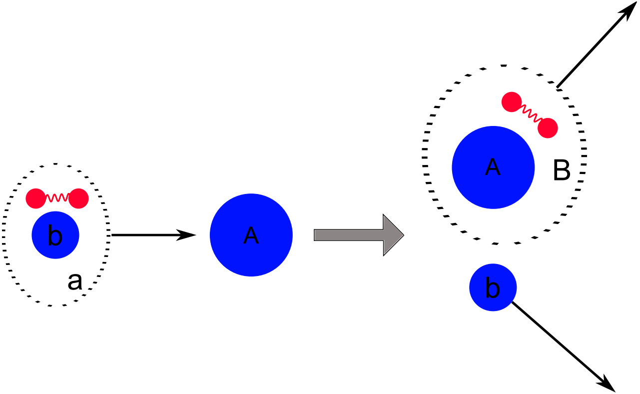
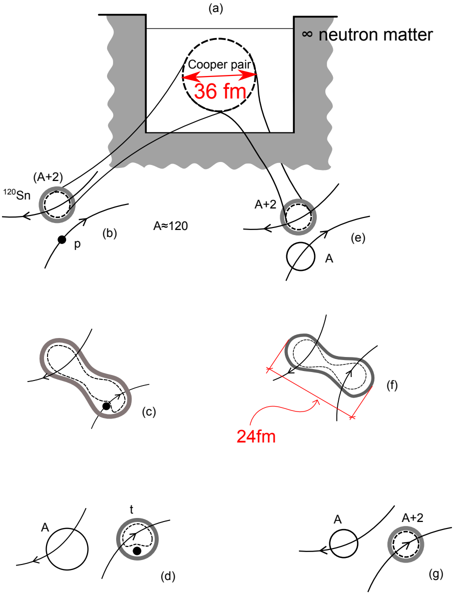
2-neutron transfer and reactions



- Reaction $A + a \longrightarrow (a - 2) + (A + 2)$.
- Measure of the pairing correlations between the transferred nucleons.
- Need to correctly account for the correlated wavefunction.

Delocalization of the pair transfer process



Let's remember the Born series

$$|\phi\rangle = |\phi_0\rangle + G_0 V |\phi\rangle$$

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$$|\phi\rangle = |\phi_0\rangle + G_0 V \left(|\phi_0\rangle + G_0 V|\phi\rangle\right)$$

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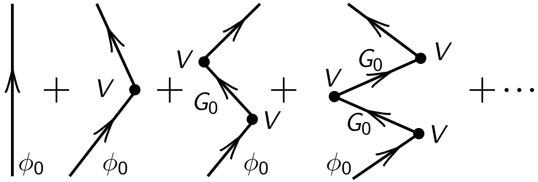
$$|\phi\rangle = |\phi_0\rangle + G_0 V \left(|\phi_0\rangle + G_0 V \left[|\phi_0\rangle + G_0 V|\phi\rangle\right]\right)$$

Let's remember the Born series

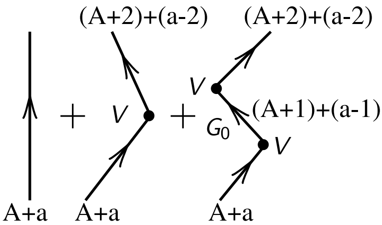
$$|\phi\rangle = |\phi_0\rangle + G_0 V \phi_0 + G_0 V G_0 V \phi_0 + G_0 V G_0 V G_0 V \phi_0 + \ldots$$



Two-nucleon transfer: stick to second order

In order to account for the successive transfer of two nucleons, we stick to 2nd order, $A + a \quad \rightarrow \quad (A+1) + (a-1) \quad \rightarrow \quad (A+2) + (a-2)$

$$|\phi\rangle = |\phi_0\rangle + G_0 V \phi_0 + G_0 V G_0 V \phi_0$$

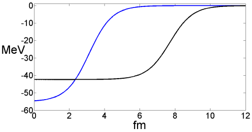


- $V \rightarrow$ one-nucleon transfer
- $G_0 \rightarrow$ Green's function of each one of the many intermediate states $(A+1) + (a-1)$

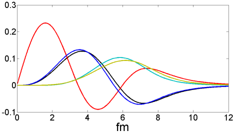
Structure:

$$\Phi_i(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2)=\sum_{j_i} B_{j_i}\left[\psi^{j_i}(\mathbf{r}_1,\sigma_1)\psi^{j_i}(\mathbf{r}_2,\sigma_2)\right]_0^0$$

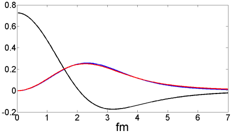
$$\Phi_f(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2)=\sum_{j_f} B_{j_f}\left[\psi^{j_f}(\mathbf{r}_1,\sigma_1)\psi^{j_f}(\mathbf{r}_2,\sigma_2)\right]_0^0$$



mean field potentials



radial wave functions $u^{j_i}(r)$



radial wave functions $u^{j_f}(r)$

Reaction:

$$T_{2NT}=\sum_{j_f j_i} B_{j_f}B_{j_i}\left(T^{(1)}(j_i,j_f)+T^{(2)}_{succ}(j_i,j_f)-T^{(2)}_{NO}(j_i,j_f)\right)$$

Introducing $T^{(1)}(j_i, j_f)$, $T^{(2)}_{succ}(j_i, j_f)$ and $T^{(2)}_{NO}(j_i, j_f)$

very schematically, the first order (*simultaneous*) contribution is

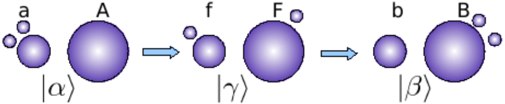
$$T^{(1)} = \langle \beta | V | \alpha \rangle,$$

while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

$$\begin{aligned} T^{(2)} &= T^{(2)}_{succ} + T^{(2)}_{NO} \\ &= \sum_\gamma \langle \beta | V | \gamma \rangle G \langle \gamma | V | \alpha \rangle - \sum_\gamma \langle \beta | \gamma \rangle \langle \gamma | V | \alpha \rangle. \end{aligned}$$

If we sum over a complete basis of intermediate states γ, we can apply the closure condition and $T^{(2)}_{NO}$ cancels $T^{(1)}$

the transition potential being single particle, two-nucleon transfer is a second order process.

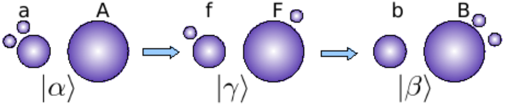


Potel *et al.*, PRL **107** 092501 (2011)
Potel *et al.*, PRL **105** 172502 (2010)

$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} \, d\mathbf{r}_{b1} \, d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{bB}(\mathbf{r}_{bB})$$
$$\times \, v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]^0_0 \chi^{(+)}_{aA}(\mathbf{r}_{aA})$$



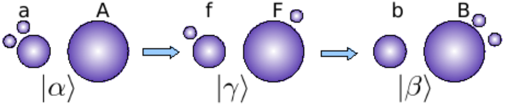
Potel *et al.*, PRL **107** 092501 (2011)
Potel *et al.*, PRL **105** 172502 (2010)

$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

Successive transfer

$$\begin{aligned} T^{(2)}_{succ}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma_1' \sigma_2'}} \int & d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \\ & \times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^K_M \\ & \times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^K_M \\ & \times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^0_0 \chi^{(+)}_{aA}(\mathbf{r}'_{aA}) \end{aligned}$$

Two particle transfer in 2–step DWBA



Potel *et al.*, PRL **107** 092501 (2011)
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$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

Non–orthogonality term

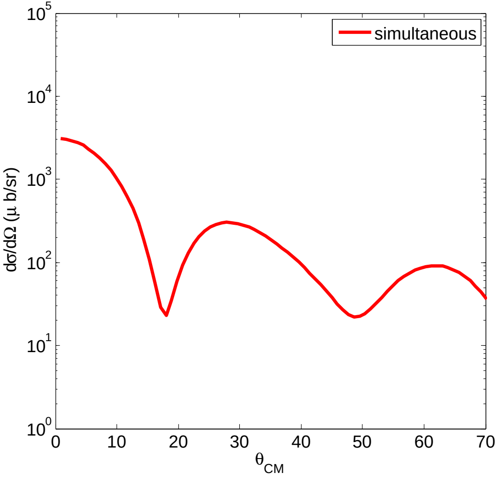
$$T^{(2)}_{NO}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma'_1\sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0$$

$$\times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^K_M$$

$$\times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^K_M$$

$$\times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^0_0 \chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

Contributions to the $^{112}Sn(p,t)^{110}$ total cross section



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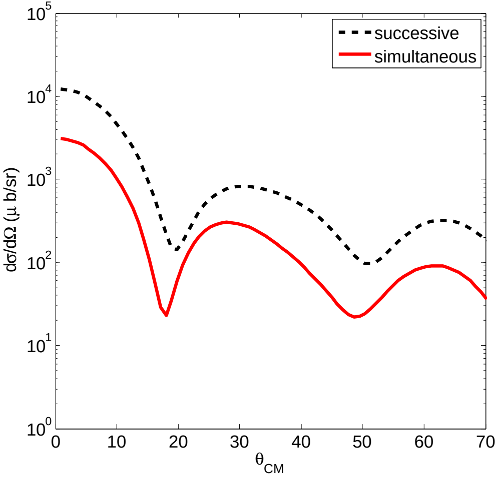

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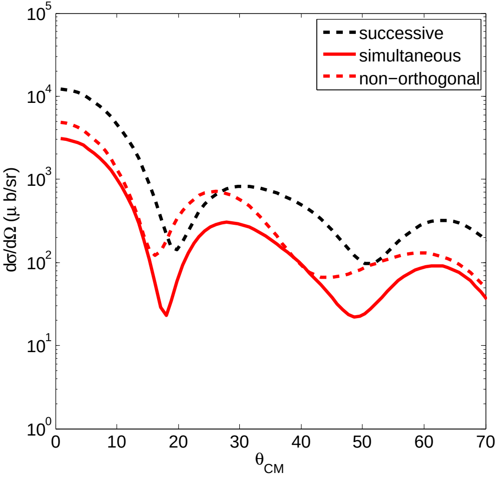
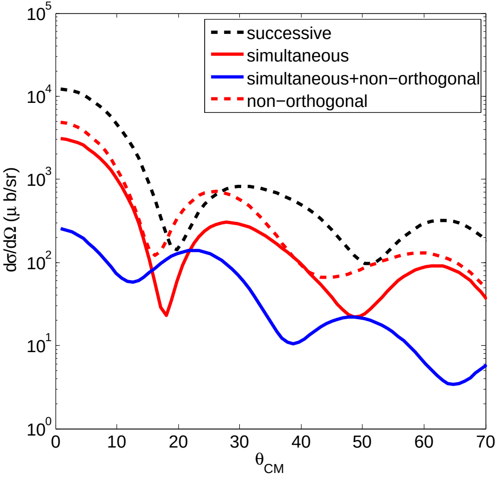


Contributions to the ^{112}Sn(p,t)^{110} total cross section



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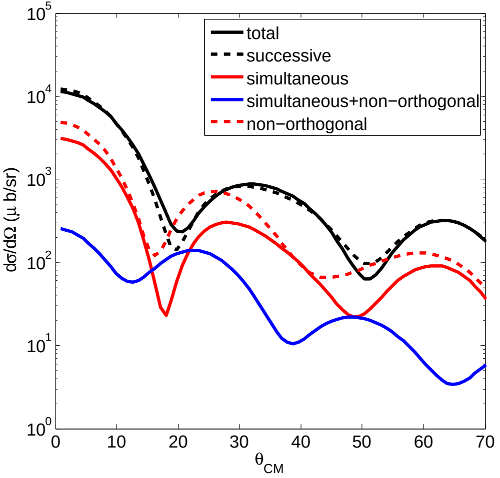
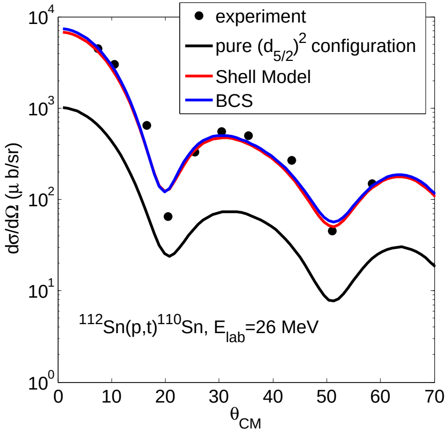


Transfer driven by single-particle potential (mean field) $\rightarrow$ essentially a successive process!

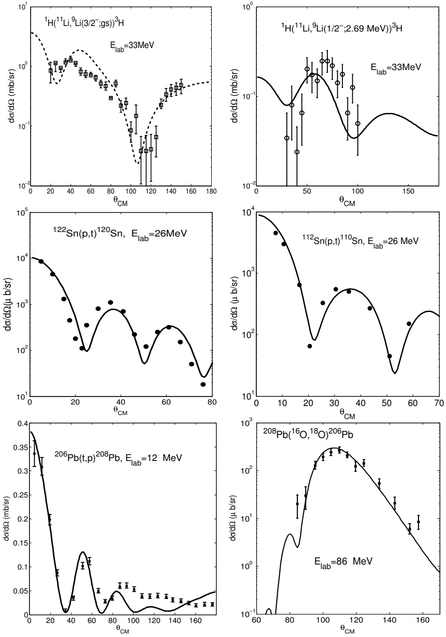


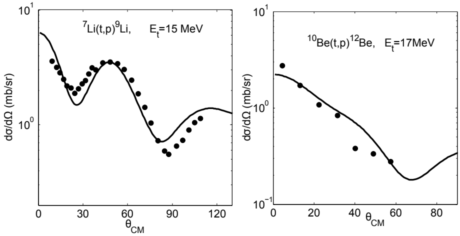
enhancement factor with respect to the transfer of uncorrelated neutrons:
$\varepsilon = 20.6$

Experimental data and shell model wavefunction from Guazzoni *et al.* PRC **74** 054605 (2006)

experiment very well reproduced with mean field (BCS) wavefunctions

Examples of calculations





good results obtained for halo nuclei,
population of excited states,
superfluid nuclei,
normal nuclei (pairing vibrations),
heavy ion reactions...
Potel *et al.*, Rep. Prog. Phys. **76** (2013) 106301

Absolute cross sections reproduced

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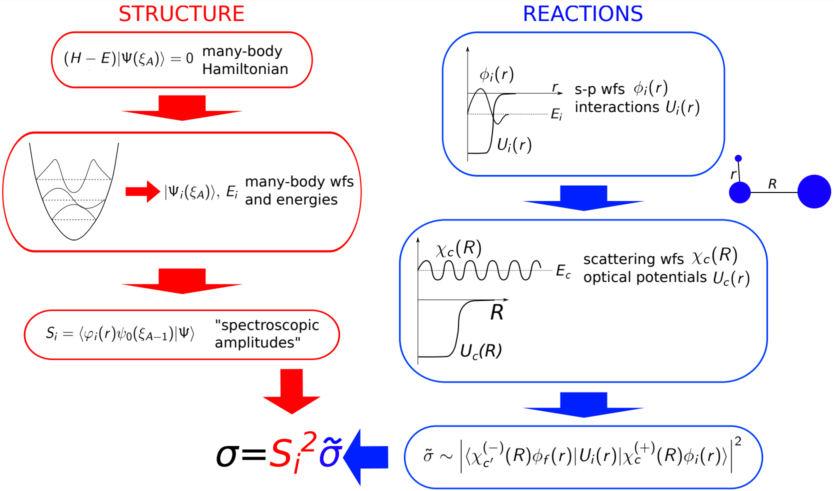
Extracting the structure information: a standard approach



- Factorization of structure and reactions.
- Can suffer from inconsistency between the two schemes.
- Extracted spectroscopic factor $S_i^2 = \sigma/\tilde{\sigma}$ problematic.

(d,p) reactions: a unified approach

1. Describe the structure of the 2-body subsystems in some given framework of choice.
2. Employ the same quantum many-body methods to work out the interactions U_{An}, U_{Ap}, U_{pn} (in general, non-local and energy-dependent).
3. Write down the resulting 3–body Hamiltonian H.
4. Obtain cross sections from H using controlled approximations.

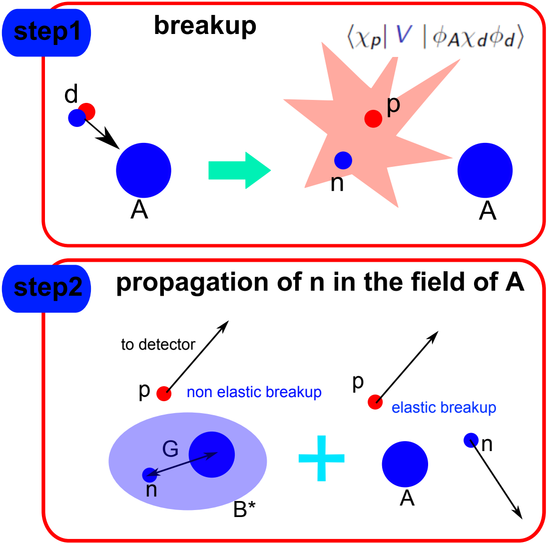
3–body Hamiltonian

$$H = T + U_{An}(r_n, r'_n, E_n, J_n, \pi_n) + U_{Ap}(r_p, r'_p, E_p, J_p, \pi_p) + U_{pn}(r_{pn}, r'_{pn}, E_{pn}, J_{pn}, \pi_{pn})$$

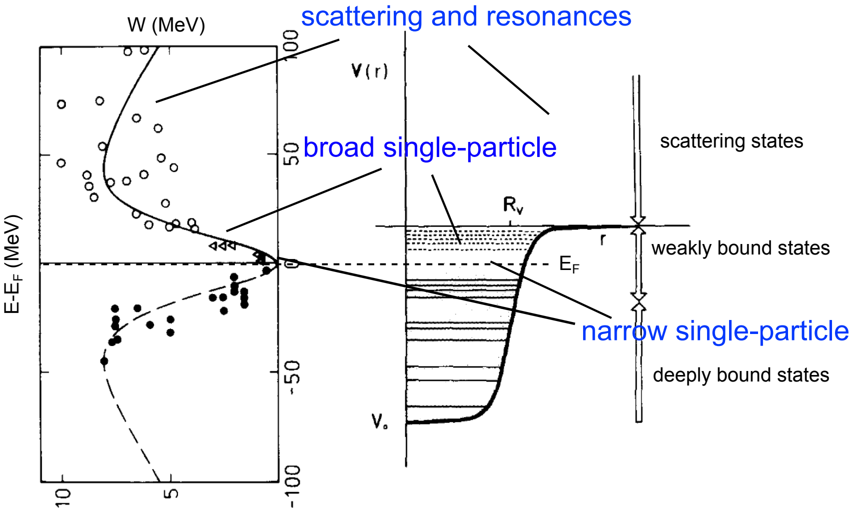
Disclaimer

Still not the end of the story! 3–body forces U_{Anp} not taken into account at this stage.

(d, p) reaction as a 2-step process

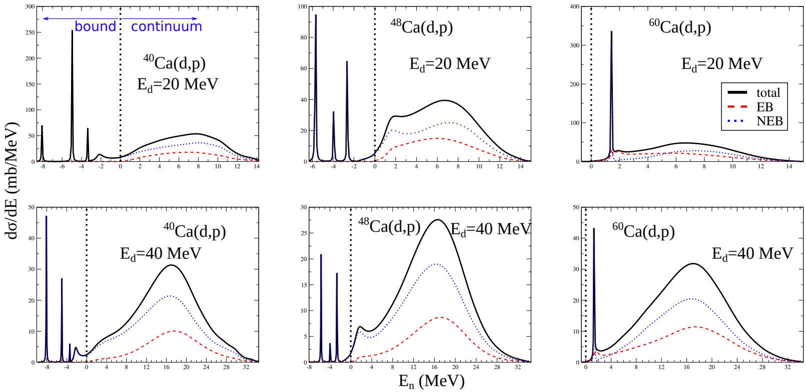


U_{An} potential and neutron states



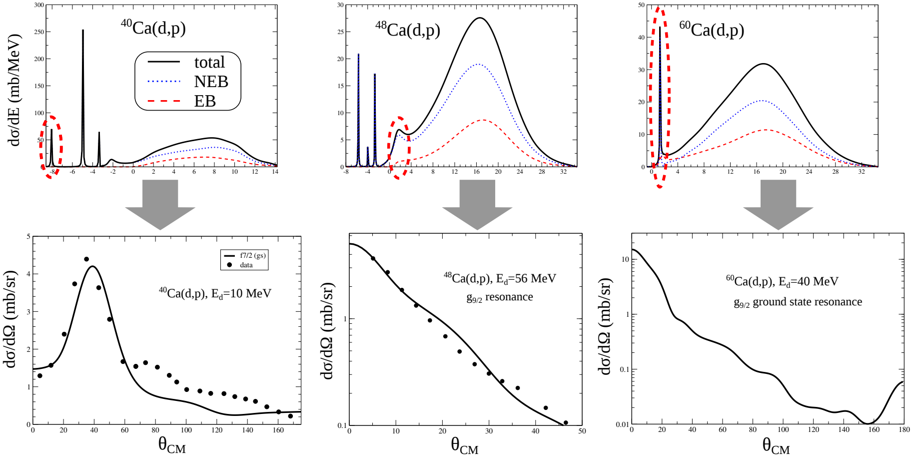
Imaginary part of optical potential neutron states

Mahaux, Bortignon, Broglia and Dasso Phys. Rep. **120** (1985) 1



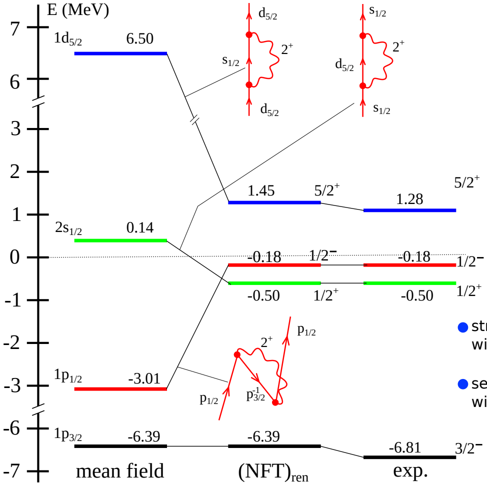
GP *et al.*, Eur. Phys. J. A **53** (2017) 178.

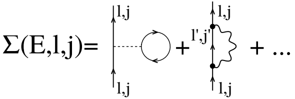
- DOM used to compute (d, p) cross sections on Ca isotopes.
- Both bound and continuum neutron states described.
- DOM can be extrapolated to unknown territory (^{60}Ca).



Absolute transfer cross sections without spectroscopic factors.

Nuclear Field Theory (NFT): ^{11}Be, parity inversion close to the dripline

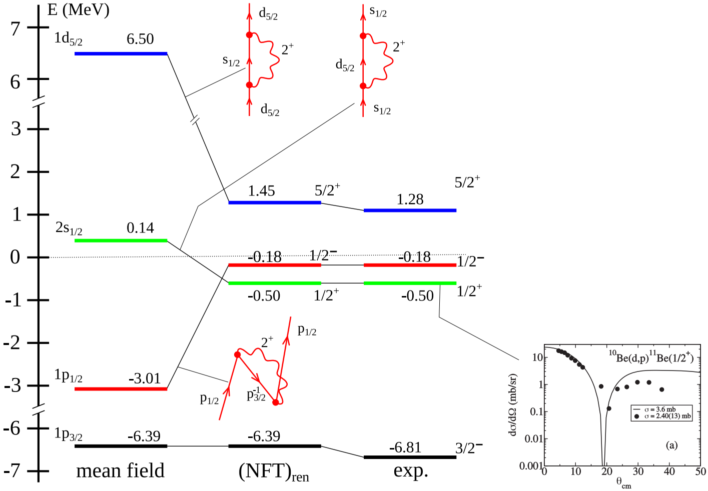




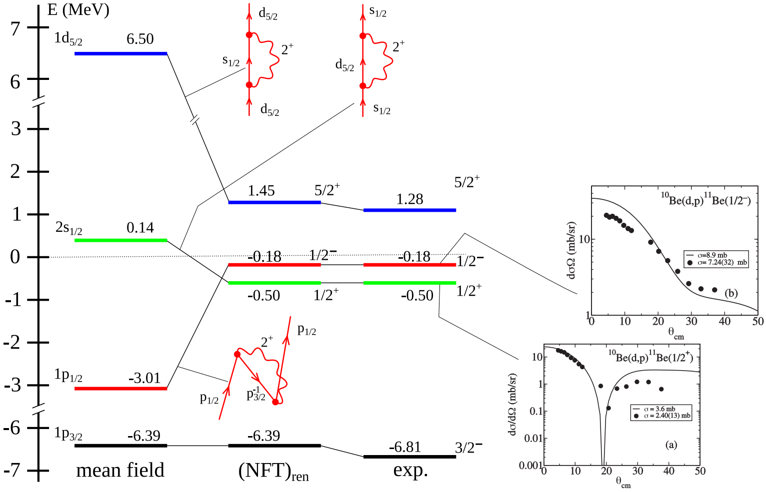
$$G(E,l,j)=G_0(E)+G_0(E)\Sigma(E,l,j)G(E,l,j)$$

- structure (energies, widths) reproduced within an EFT-like framework (NFT)
- self energies and Green's functions obtained within the same framework

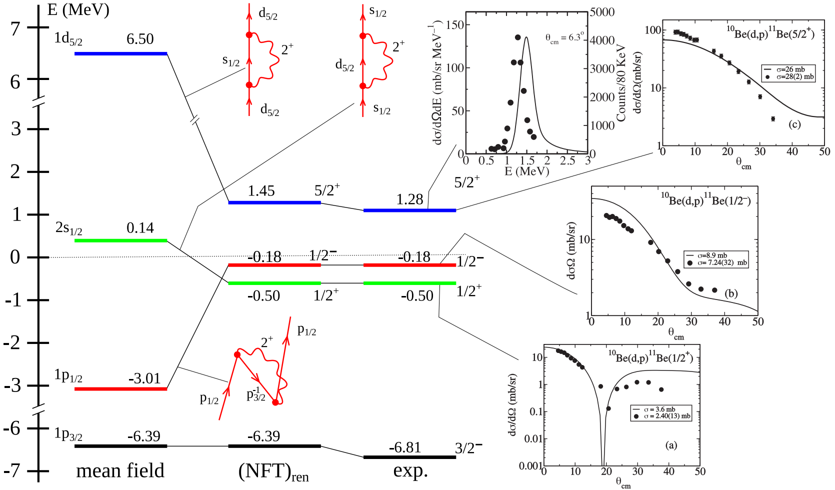
Nuclear Field Theory (NFT): ^{11}Be, parity inversion close to the dripline



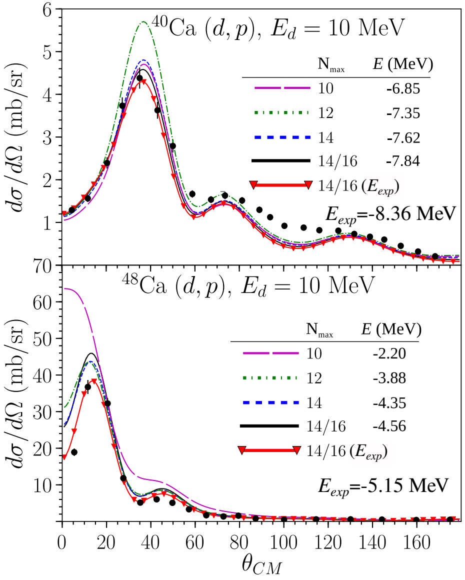
Nuclear Field Theory (NFT): ^{11}Be, parity inversion close to the dripline



Nuclear Field Theory (NFT): ^{11}Be, parity inversion close to the dripline



Barranco, GP, Broglia, Vigezzi PRL **119**, 082501 (2017)



structure calculated within *ab initio* coupled cluster framework by J. Rotureau (MSU)

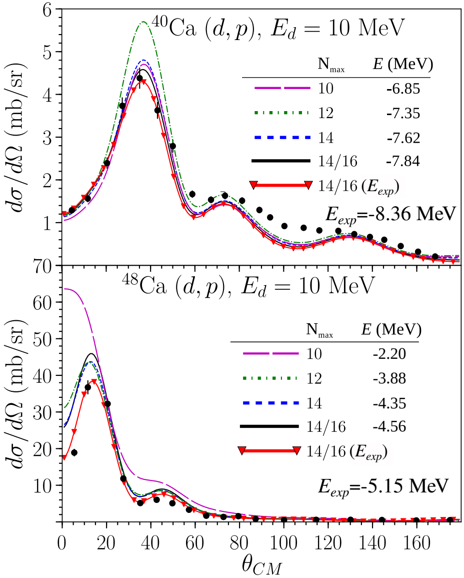
remarkable agreement with experimental data

coupled cluster+(d,p)+ continuum spectroscopy

↓

powerful tool for exotic medium-mass nuclei

Coupled-Cluster (CC): Ca isotopes



structure calculated within *ab initio* coupled cluster framework by J. Rotureau (MSU)