Nuclear Reactions I: Basics

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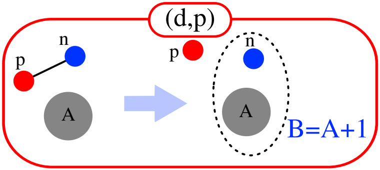
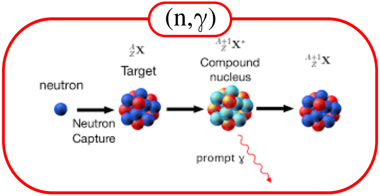
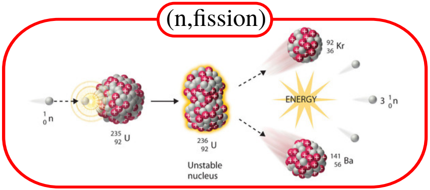
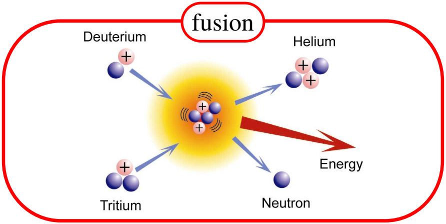
FRIB

Nuclear reactions are processes in which two nuclear species collide, allowing them to exchange matter, energy, and momentum.

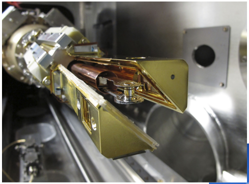






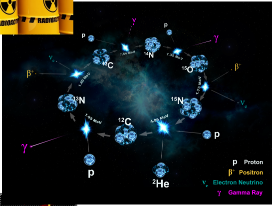


inertial confinement fusion

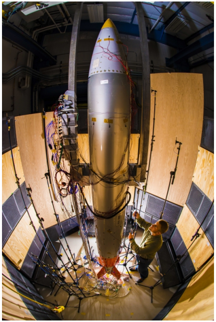


waste management

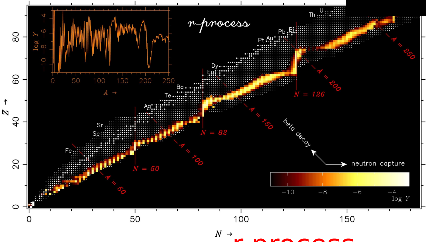
nuclear reactors



reactions of astrophysical interest



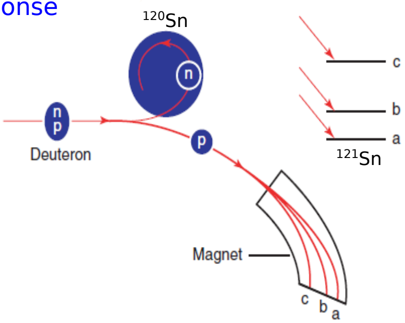
stockpile stewardship

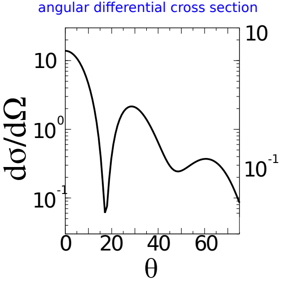


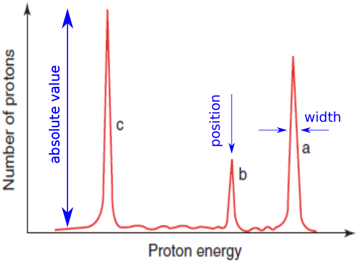
r-process

Why do we care? Nuclear reactions as an experimental tool

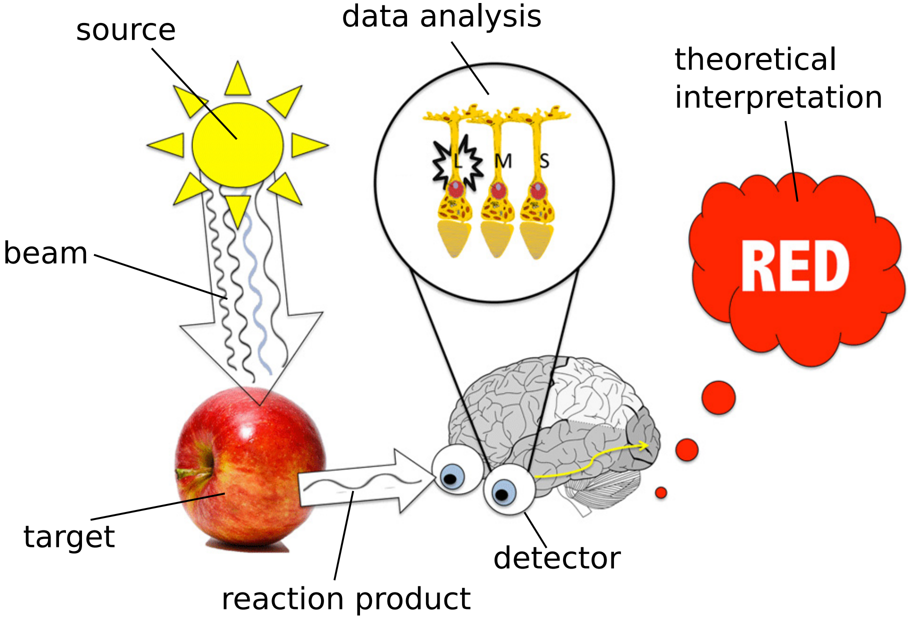
- transfer reactions probe nuclear response to the addition of a nucleon
- a variety of observables provide rich information about nuclear structure:
  - angular differential cross section
  - absolute value
  - position
  - width (when in the continuum)



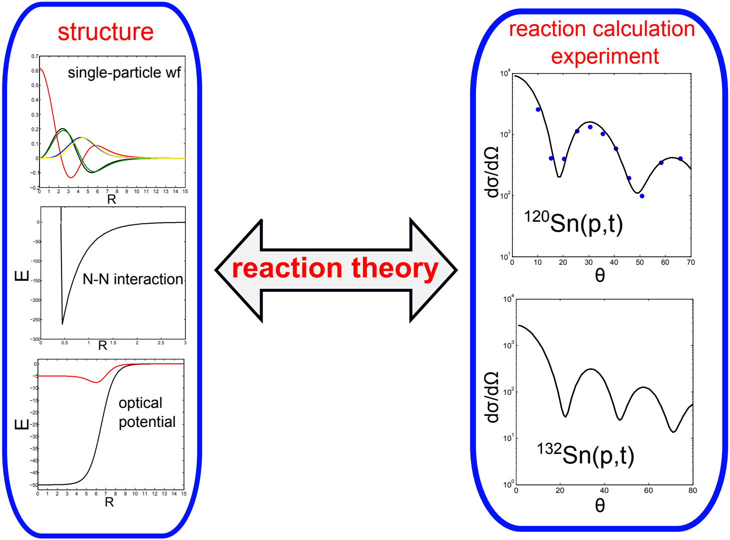




Looking at things



Reaction formalism, between structure and experiment



Time-dependent vs time-independent description

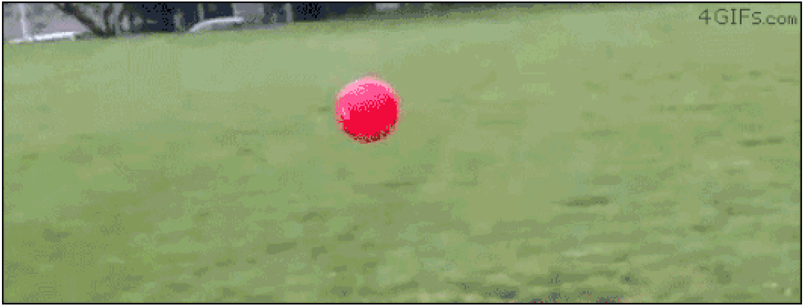
Time-dependent description $\rightarrow$ time-dependent Schrödinger equation

$$-i\hbar\frac{\partial\phi(r,t)}{\partial t} = H(r,t)\phi(r,t)$$

Time-dependent vs time-independent description

Time-dependent description → time-dependent Schrödinger equation

$$-i\hbar\frac{\partial\phi(r,t)}{\partial t} = H(r,t)\phi(r,t)$$



Time-dependent vs time-independent description

Time-independent description → time-independent Schrödinger equation

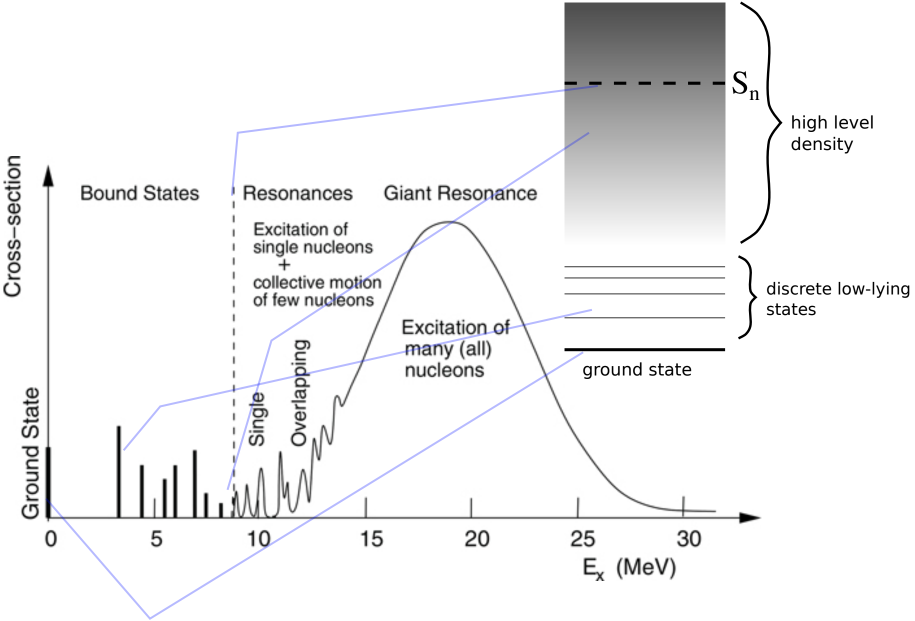
$$H(r)\phi(r) = E\phi(r)$$

Time-dependent vs time-independent description

Time-independent description $\rightarrow$ time-independent Schrödinger equation

$$H(r)\phi(r) = E\phi(r)$$

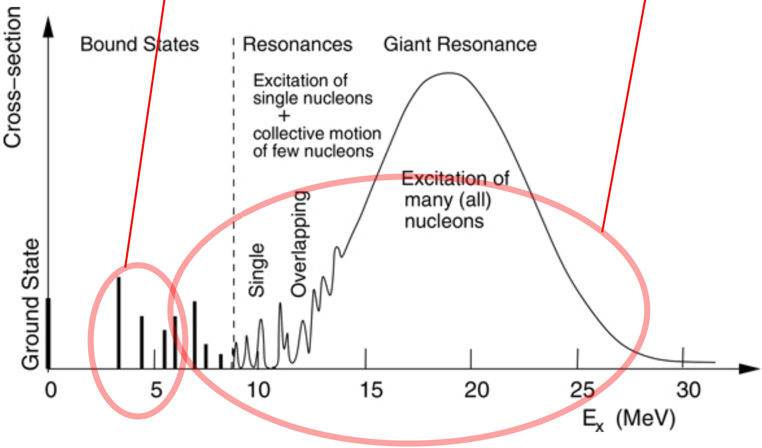
Compound and direct reactions



Compound and direct reactions

transfer, inelastic excitation, knockout

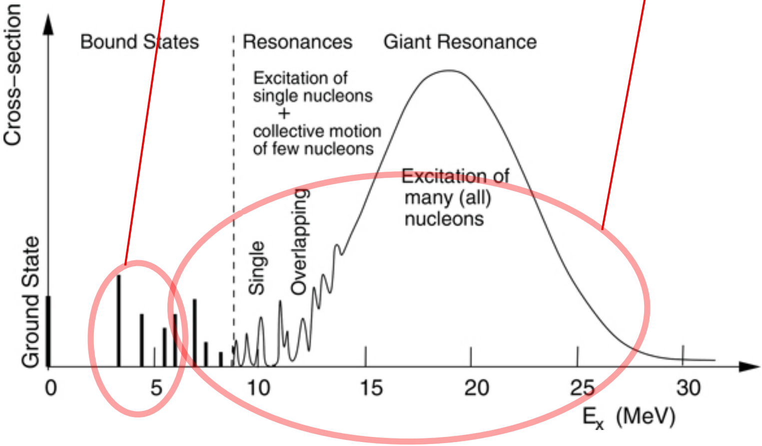
fusion, fission, compound nucleus reactions, giant resonances



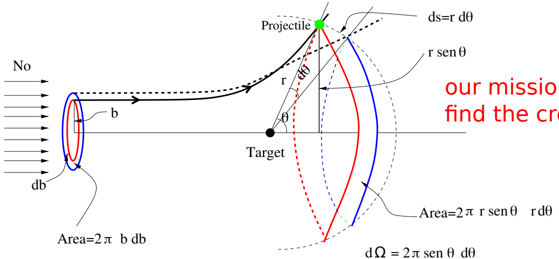
Compound and direct reactions

individual quantum states

level densities
(statistical treatment)



2-body Scrhödinger equation in 3D



our mission:
find the cross section!

$$\left(-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2}+\frac{\hbar^2\ell(\ell+1)}{2\mu r^2}+V(r)-E\right)\phi_\ell(r)=0, \quad \phi(r)=\sum_\ell \phi_\ell(r),$$

where $\hbar\ell$ is the angular momentum, related to the classical impact parameter: $\ell = kb$, where k is the linear momentum. If $V(r\to\infty)=0$,

$$\phi(r\to\infty)=e^{ikz}+\frac{e^{ikr}}{r}f(\theta)$$

$$\frac{d\sigma}{d\Omega}(\theta)=|f(\theta)|^2 \to \text{find scattering amplitude } f(\theta)!$$

Born series

Let's separate the problem into an easy and a hard to solve part,

$$H = T + U_0(r) + V(r),$$

where T is the kinetic energy operator, and $U_0(r)$ is easy to solve,

$$(T - U_0(r) - E)|\phi_0\rangle = 0.$$

$$\begin{aligned} &(T + U_0(r) - E)|\phi\rangle = -V(r)|\phi\rangle \\ &\to |\phi\rangle = |\phi_0\rangle + \lim_{\eta\to 0} \frac{1}{E - T - U_0 + i\eta} V|\phi\rangle = |\phi_0\rangle + G_0 V|\phi\rangle, \end{aligned}$$

where we have defined the operator

$$G_0 = \lim_{\eta\to 0} \frac{1}{E - T - U_0 + i\eta}$$

called the Green's function, a.k.a. propagator.

(Interlude: the Green's function)

The Green's function is a non-local, integral operator

$$G_0|\phi\rangle \equiv \int G_0(r,r')\phi(r')\,dr',$$

with

$$G_0(r,r') = \frac{1}{k}\begin{cases} \chi(r)F(r') & r < r' \\ \chi(r')F(r) & r > r' \end{cases}$$

χ and F are two linearly independent solutions of the Hamiltonian $T + U_0$,

$$(T + U_0 - E)\chi(r) = 0, \quad (T + U_0 - E)F(r) = 0,$$

with the boundary conditions,

$$\chi(r \to 0) = 0, \quad F(r \to \infty) = \frac{e^{ikr}}{r}$$

Born series

$$|\phi\rangle = |\phi_0\rangle + G_0 V |\phi\rangle$$

Born series

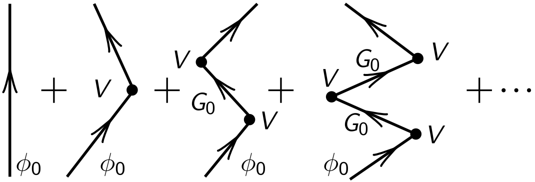
$$|\phi\rangle = |\phi_0\rangle + G_0 V \left(|\phi_0\rangle + G_0 V|\phi\rangle\right)$$

Born series

$$|\phi\rangle = |\phi_0\rangle + G_0 V \left(|\phi_0\rangle + G_0 V \left[|\phi_0\rangle + G_0 V |\phi\rangle\right]\right)$$

Born series

$$|\phi\rangle = |\phi_0\rangle + G_0 V \phi_0 + G_0 V G_0 V \phi_0 + G_0 V G_0 V G_0 V \phi_0 + \dots$$



Born series

$$|\phi\rangle = |\phi_0\rangle + G_0 V|\phi_0\rangle$$

The first order term is known as the first-order Distorted Wave Born Approximation (DWBA)

$$|\phi\rangle \approx |\phi_0\rangle + G_0 V|\phi_0\rangle.$$

In this approximation,

$$\begin{aligned}\phi(r) = \phi_0(r) &+ \int G_0(r, r')V(r')\phi_0(r')\,dr' \\ &= \phi_0 + \frac{1}{k}\begin{cases} \chi(r)\int F(r')V(r')\phi_0(r')\,dr' & r < r' \\ F(r')\int \chi(r')V(r')\phi_0(r')\,dr' & r > r' \end{cases}\end{aligned}$$

$$\phi(r \to \infty) = e^{ikz} + \frac{e^{ikr}}{r}\int \chi(r')V(r')\phi_0(r')\,dr'$$

Scattering ampitude in DWBA

Remember:

$$\phi(r \to \infty) = e^{ikz} + \frac{e^{ikr}}{r} f(\theta).$$

In first-order DWBA

$$\phi(r \to \infty) = e^{ikz} + \frac{e^{ikr}}{r} \int \chi(r') V(r') \phi_0(r')\, dr'$$

so, in DWBA, the scattering amplitude (a.k.a T matrix) is

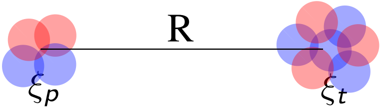
$$T(\theta) \equiv f(\theta) = \int \chi(r') V(r') \phi_0(r')\, dr' \equiv \langle \chi | V | \phi_0 \rangle$$

We're done!

The desired cross section is

$$\frac{d\sigma}{d\Omega}(\theta) = |T(\theta)|^2$$

Nuclei are composite objects: beyond 2-bodies



Target and projectile have internal degrees of freedom $\xi \equiv \{\xi_t, \xi_p\}$.

$$H = T_R + h(\xi) + V(\xi, R)$$
$$(h - \epsilon)\,\phi(\xi) = 0, \quad \phi(\xi) = \phi_t(\xi_t)\phi_p(\xi_p)$$

$$\Psi(\xi, R) = \sum_i \phi_i(\xi)\chi_i(R)$$

The index i runs through all the channels. The scattering wavefunction $\chi_i(R)$ brings to the detector the information that channel i has been populated. The elastic channel 0 is the only one present as a plane wave,

$$\Psi(\xi, R \to \infty) = \phi_0(\xi)e^{ik_0 R_z} + \sum_i \frac{e^{ik_i R}}{R}\,\phi_i(\xi)f_i(\theta)$$

Example: 2-channel system

$$\Psi(\xi, R) = \phi_0(\xi)\chi_0(R) + \phi_1(\xi)\chi_1(R)$$

with

$$(h - \epsilon_0)\,\phi_0(\xi) = 0, \quad (h - \epsilon_1)\,\phi_1(\xi) = 0.$$

The Schrödinger equation

$$(T_R + V(\xi, R) + h(\xi) - E)|\Psi\rangle = 0$$

can be projected on channels 0,

$$\begin{aligned} &\langle\phi_0|(T_R + V(\xi, R) + h(\xi) - E)|\Psi\rangle = 0 \\ &\to (T_R + \langle\phi_0|V|\phi_0\rangle + \epsilon_0 - E)\chi_0(R) = -\langle\phi_0|V|\phi_1\rangle\chi_1(R), \end{aligned}$$

and 1,

$$\begin{aligned} &\langle\phi_1|(T_R + V(\xi, R) + h(\xi) - E)|\Psi\rangle = 0 \\ &\to (T_R + \langle\phi_1|V|\phi_1\rangle + \epsilon_1 - E)\chi_1(R) = -\langle\phi_1|V|\phi_0\rangle\chi_0(R), \end{aligned}$$

If we define

$$E_i = E - \epsilon_i, \qquad V_{ij}(R) = \langle \phi_i | V | \phi_j \rangle \equiv \int \phi_i(\xi) V(\xi, R) \phi_j(\xi) \, d\xi,$$

we obtain the coupled equations

$$(T_R + V_{00}(R) - E_0)\chi_0(R) = -V_{01}\chi_1(R),$$
$$(T_R + V_{11}(R) - E_1)\chi_1(R) = -V_{10}\chi_0(R).$$

From the second equation,

$$\chi_1(R) = \lim_{\eta \to 0} (E_1 - T_R - V_{11} + i\eta)^{-1} V_{10}\chi_0(R) = G_1 V_{10}\chi_0(R).$$

We can substitute in the first equation,

$$(T_R + V_{00}(R) - E_0)\chi_0(R) = -V_{01} G_1 V_{10}\chi_0(R)$$

Coupled equations and optical potential

We can rewrite

$$(T_R + U_0(R) - E_0)\chi_0(R) = 0,$$

where

$$U_0(R) = V_{00} + V_{01} G_1 V_{10}$$

is the optical potential. For an arbitrary number of channels,

$$(T_R + V_{ii}(R) - E_i)\chi_i(R) = -\sum_{j \neq i} V_{ij}\chi_j(R),$$

and the optical potential is

$$U_0(R) = V_{00} + \sum_{i \neq 0} V_{0i} G_i V_{i0},$$

where

$$G_i = \lim_{\eta \to 0} (E_i - T_R - V_{ii} + i\eta)^{-1},$$

is the Green's function in the channel i.

Solving the coupled equations: first order DWBA

Let's compute the cross section for the population of channel c. We'll do that assuming that all channels are weakly populated, so the elastic channel 0 strongly dominates.

Solving the coupled equations: first order DWBA

1. Get states and interactions (input from structure!),

$$V_{ij}(R) = \langle \phi_i | V | \phi_j \rangle \equiv \int \phi_i(\xi) V(\xi, R) \phi_j(\xi)\, d\xi,$$

2. Get the elastic channel,

$$(T_R + V_{00} - E_0)\chi_0 = -\sum_{i \neq 0} V_{0i}\chi_i \to (T_R + V_{00} - E_0)\chi_0 = 0.$$

3. Write down channel c,

$$(T_R + V_{cc} - E_c)\chi_c = -\sum_{i \neq c} V_{ci}\chi_i \to (T_R + V_{cc} - E_c)\chi_c = V_{c0}\,\chi_0$$

4. Solve using $G_c = \lim_{\eta \to 0}(E_c - T - V_{cc} + i\eta)$

$$\chi_c(R) = G_c V_{c0}\,\chi_0(R) = \int G_c(R, R') V_{c0}(R') \chi_0(R')\, dR'$$

5. Get the amplitude where the detector is $(R \to \infty)$,

$$\chi_c(R \to \infty) = \frac{e^{k_c R}}{R} \int \chi_c(R') V_{c0}(R') \chi_0(R')\, dR'$$

Solving the coupled equations: first order DWBA

We're done!

Remember that

$$(T_R + V_{cc} - E_c)\chi_c = 0.$$

We now have the T matrix,

$$\begin{aligned} T_c &= \int \chi_c(R')V_{c0}(R')\chi_0(R')\, dR' \\ &= \int \chi_c(R')\phi_c(\xi)V(R',\xi)\phi_0(\xi)\chi_0(R')\, dR'\, d\xi. \end{aligned}$$

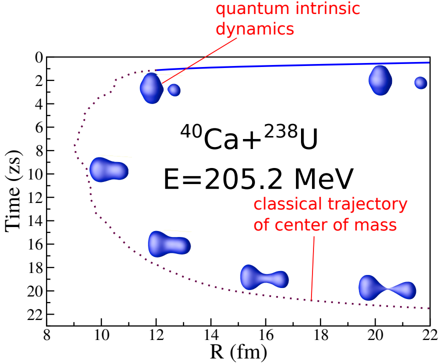
Note the parallel with Fermi's Golden rule! Now we can write down the desired cross section,

$$\frac{d\sigma}{d\Omega}(\theta) = |T_c(\theta)|^2$$

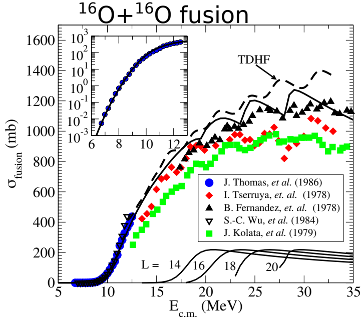
In the Time Dependent Hartree Fock (TDHF) method, the time-dependent Schrödinger equation is solved, with a mean-field Hamiltonian H,

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = H\Psi(t),$$

and $\Psi(t)$ is obtained as a function of time.



C. Simenel, S. Umar

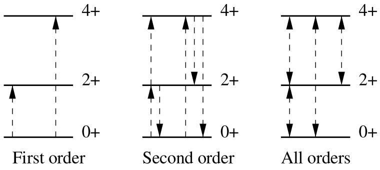


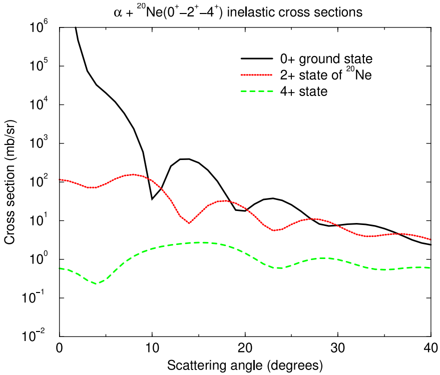
The coupled differential equations are solved to all orders, with a limited number of channels M,

$$(T_R + V_{ii}(R) - E_i)\chi_i(R) = -\sum_{j\neq i}^{M} V_{ij}\chi_j(R),$$

I. Thompson

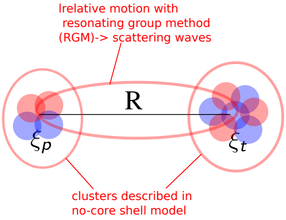
α-particle scattering on ^{20}Ne.





Other approaches: microscopic methods

- Interacting clusters described with a microscopic Hamiltonian with 2- and 3-body forces (no-core shell model).
- Relative motion described with scattering waves.
- Relative motion and coupled to intrinsic dynamics.
- computationally expensive: only light ions.



P. Navrátil, S. Quaglioni