INSTITUTE FOR STRUCTURE AND NUCLEAR ASTROPHYSICS

JINA–CEE

Beam, Spectrometer, Separator …

Manoel Couder

University of Notre Dame

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Goals

- Getting familiar with typical ion optics formalism
- Use it to define quantities that are used in beam tuning, spectrometer and separator specifications

What to Remember

- Want to get best beam ever
  - Speak with your operators and speak their vocabulary
  - Ask them questions
- Your experiments depends on beams, spectrometers, separators
  - The ability to calculate trajectories and tune is a big advantage
  - Talk to people to get going with such calculations
    - Large group at MSU/NSCL/FRIB and ANL/ATLAS
    - Smaller group scattered around the country
    - Dedicated accelerator groups at all the national labs

Good Aim is Sometime Important

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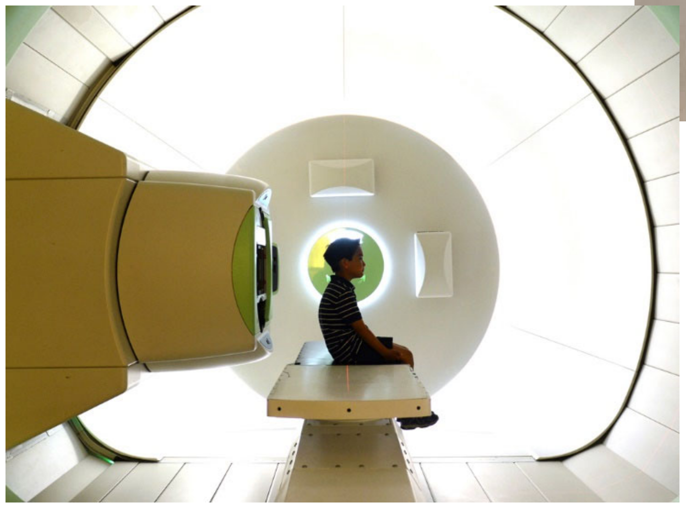
ISNAP

Sometime it is Critical!

External beam radiation targeted for prostate cancer treatment



We Will Not Talk About Accelerator Technology

World wide inventory of accelerators, in total 15,000. The data have been collected by W. Scarf and W. Wiesczycka (See U. Amaldi Europhysics News, June 31, 2000)	
Category	Number
Ion implanters and surface modifications	7,000
Accelerators in industry	1,500
Accelerators in non-nuclear research	1,000
Radiotherapy	5,000
Medical isotopes production	200
Hadron therapy	20
Synchrotron radiation sources	70
Nuclear and particle physics research	110

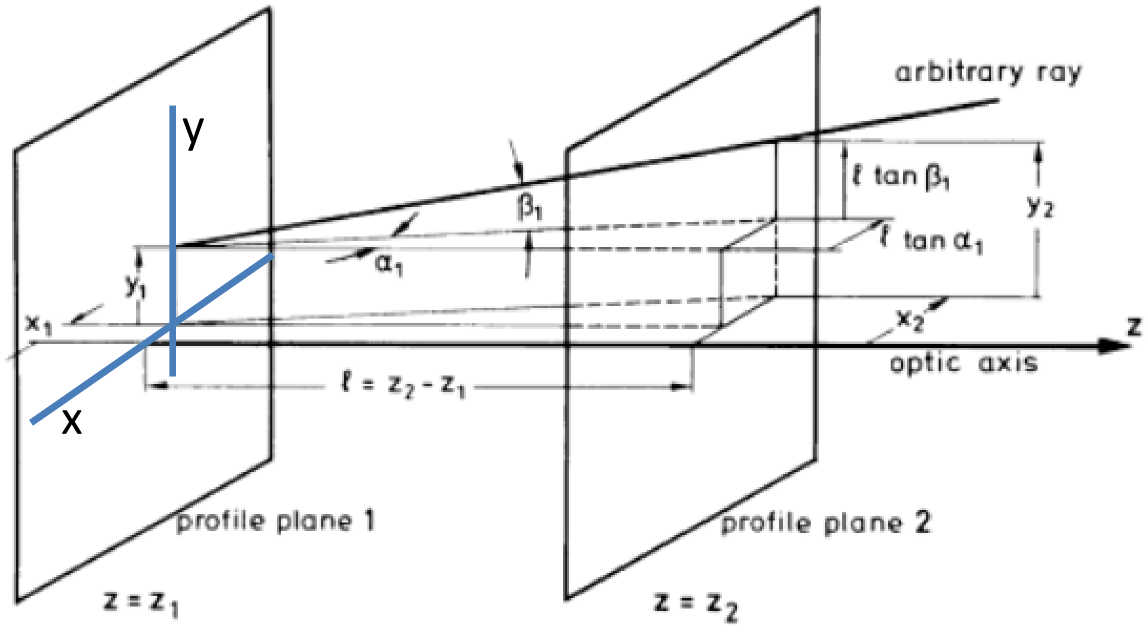
Outline

- Optics
  - Formalism
- Charged particle optics
  - Dispersion and spectrometer/separator resolving power
- Beam
  - Collection description of particles

Straight Light (Photons) Rays

- Z axis = Optics axis of a bundle of rays
- Deviation of rays from bundle

$$x(z_2) = x_1 + (z_2 - z_1)\tan(\alpha_1)$$
$$y(z_2) = y_1 + (z_2 - z_1)\tan(\beta_1)$$



Will be using this a lot!!!

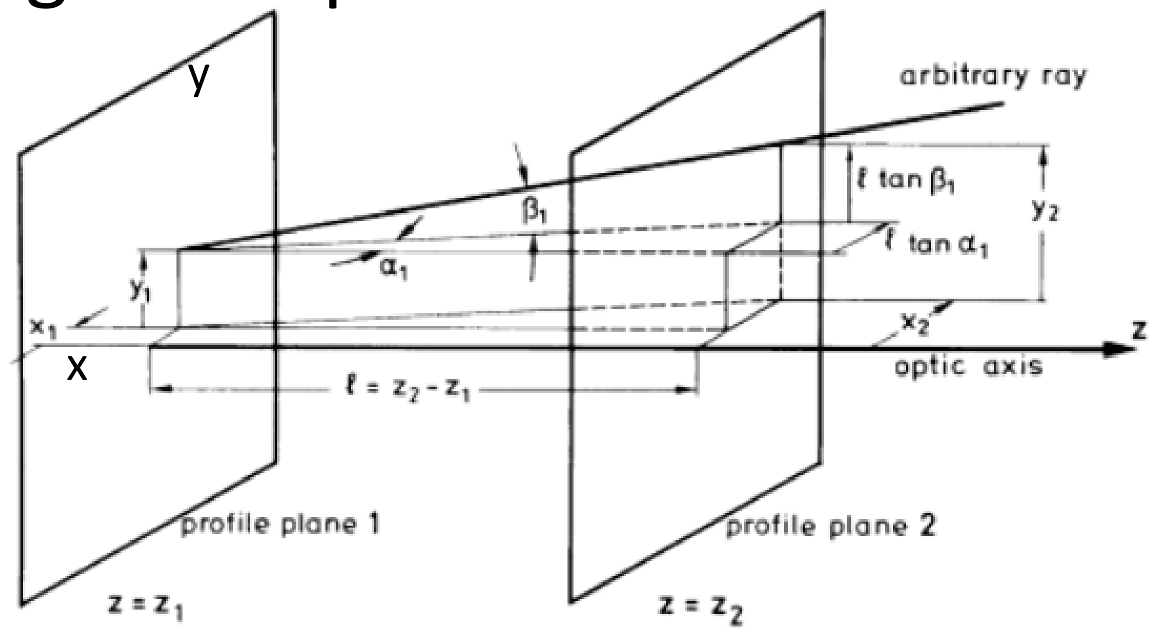
Optics of Charged Particles
By Hermann Wollnik

Straight Light (Photons) Rays

- Z axis = Optics axis of a bundle of rays
- Deviation of rays from bundle
- Angular dependence

$$x(z_2) = x_1 + (z_2 - z_1)\tan(\alpha_1)$$
$$y(z_2) = y_1 + (z_2 - z_1)\tan(\beta_1)$$

$$\tan(\alpha(z)) = \tan(\alpha_1)$$
$$\tan(\beta(z)) = \tan(\beta_1)$$



Optics of Charged Particles
By Hermann Wollnik

Not really exciting …

Straight Light (Photons) Rays

- Z axis = Optics axis of a bundle of rays
- Deviation of rays from bundle
- Angular dependence

$$x(z_2) = x_1 + (z_2 - z_1)\tan(\alpha_1) \qquad \tan(\alpha(z)) = \tan(\alpha_1)$$
$$y(z_2) = y_1 + (z_2 - z_1)\tan(\beta_1) \qquad \tan(\beta(z)) = \tan(\beta_1)$$

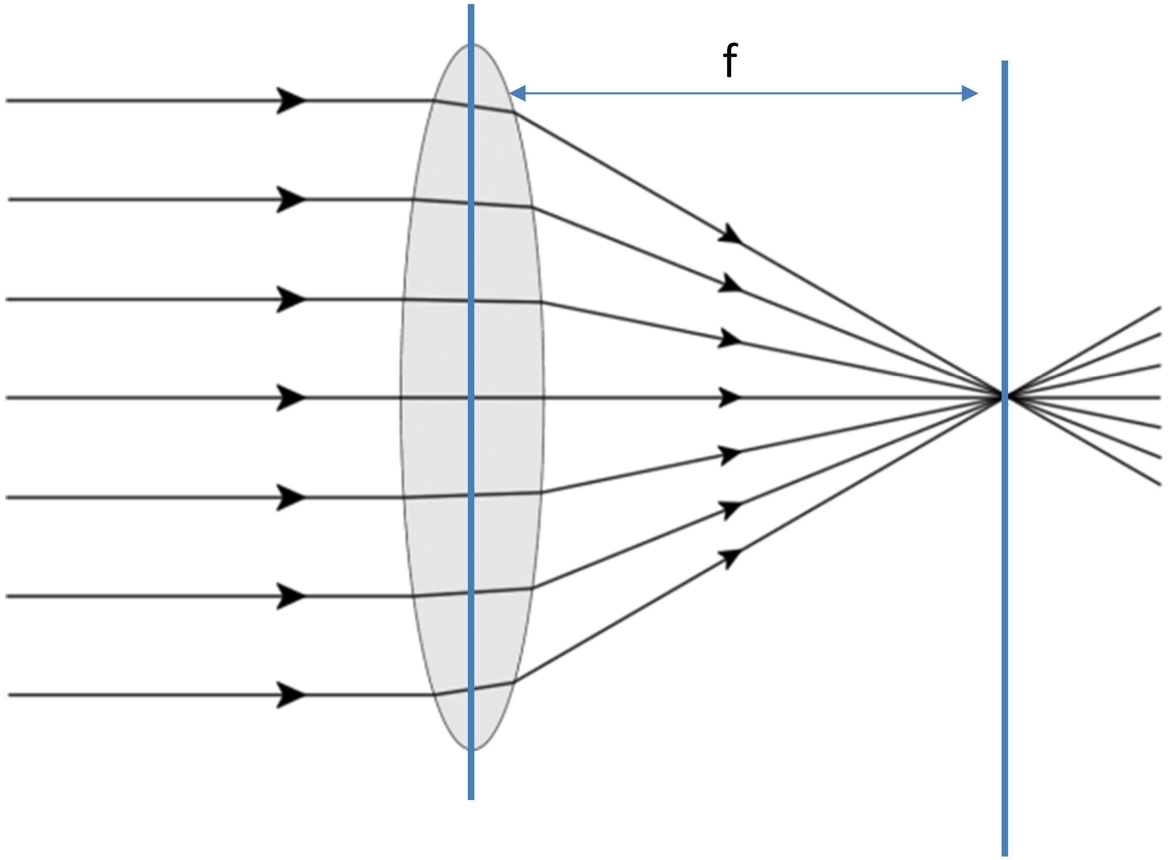
Transfer matrix from one profile plane to another

$$\begin{pmatrix} x_2 \\ \tan(\alpha_2) \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \tan(\alpha_1) \end{pmatrix}$$
$$\begin{pmatrix} y_2 \\ \tan(\beta_2) \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ \tan(\beta_1) \end{pmatrix}$$

In rotationally symmetric system
Both matrix are identical…

Thin Lens

- Rays that enter a system parallel to the optical axis are focused if that they pass through a single point.
- https://phet.colorado.edu/en/simulation/legacy/geometric-optics
- Any ray that passes through that point will emerge from the system parallel to the optical axis.



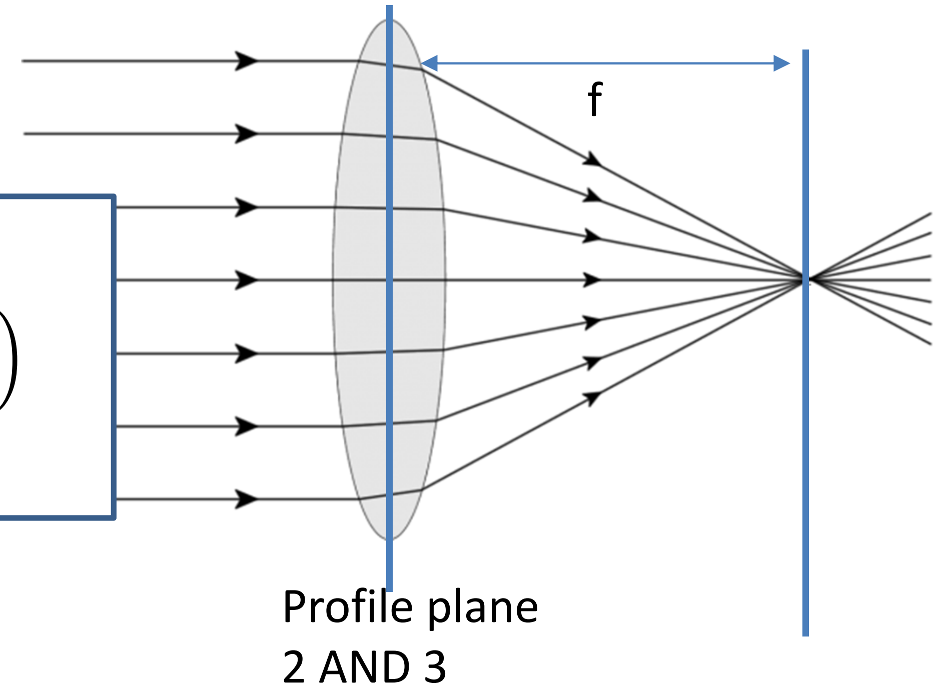
Thin Lens

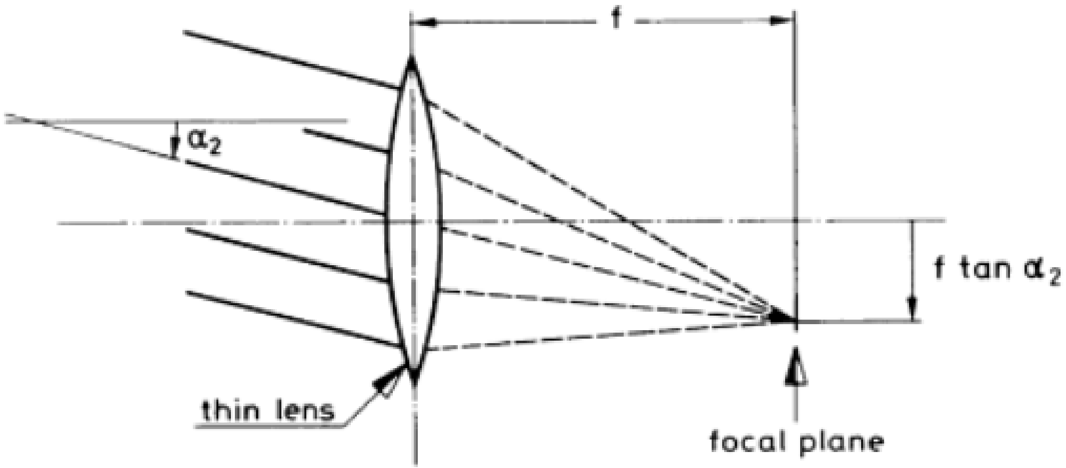
- transfer from 2 to 3

$x_3 = x_2$

$\tan(\alpha_3) = -x_2/f$

$$\begin{pmatrix} x_3 \\ \tan(\alpha_3) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ \tan(\alpha_2) \end{pmatrix}$$

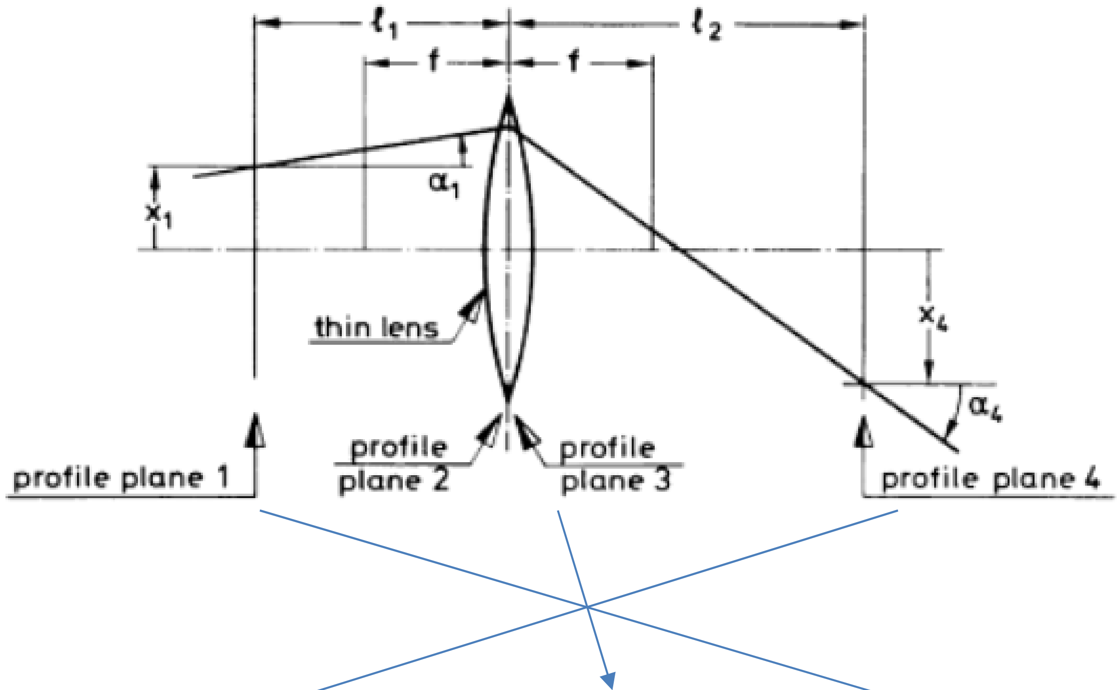




$x_3 = x_2$

$\tan(\alpha_3) = \tan(\alpha_2) - x_2/f$

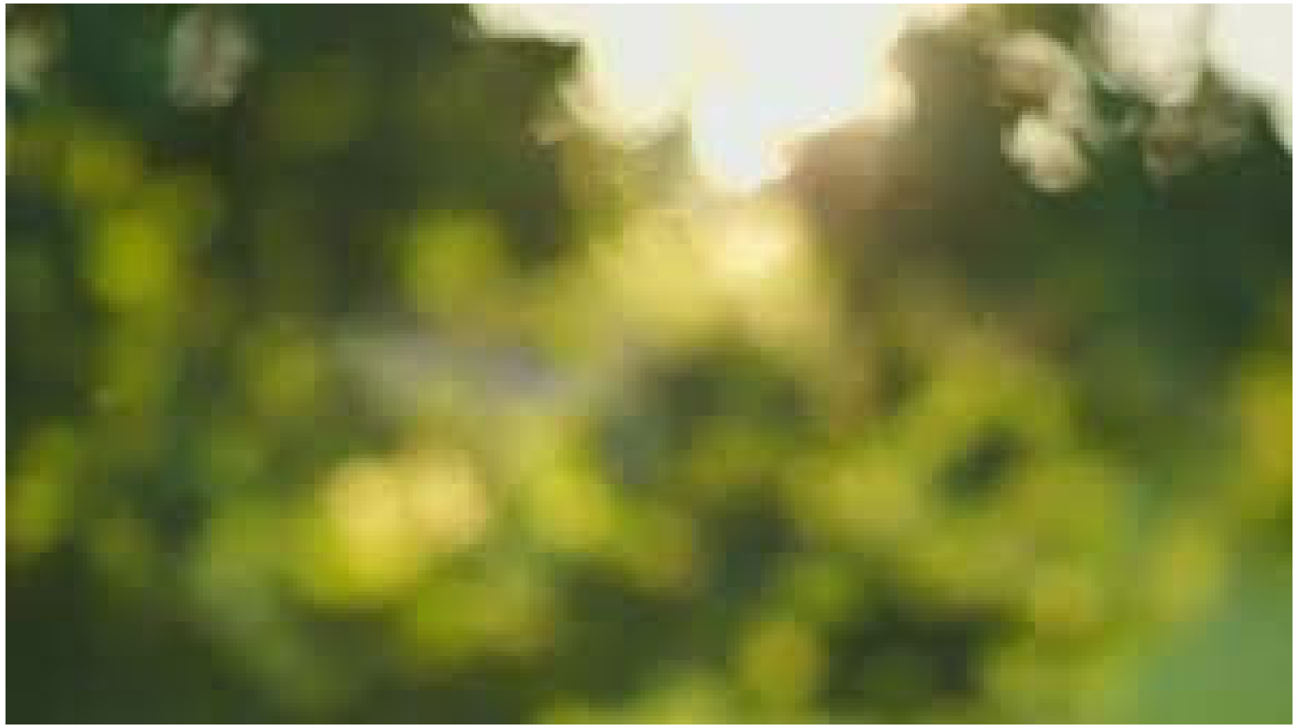
Transport through…



$$\begin{pmatrix} x_4 \\ \tan(\alpha_4) \end{pmatrix} = \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \tan(\alpha_1) \end{pmatrix}$$

$$\begin{pmatrix} x_4 \\ \tan(\alpha_4) \end{pmatrix} = \begin{pmatrix} 1-(l_2/f) & l_1+l_2-(l_1 l_2/f) \\ -1/f & 1-(l_1/f) \end{pmatrix} \begin{pmatrix} x_1 \\ \tan(\alpha_1) \end{pmatrix}$$

How do we Get a Proper Picture?



$$\begin{pmatrix} x_4 \\ \tan(\alpha_4) \end{pmatrix} = \begin{pmatrix} 1-(l_2/f) & l_1+l_2-(l_1 l_2/f) \\ -1/f & 1-(l_1/f) \end{pmatrix} \begin{pmatrix} x_1 \\ \tan(\alpha_1) \end{pmatrix}$$

Independence of final ray position on initial angle

$$l_1 + l_2 - (l_1 l_2/f) = 0$$

$$\downarrow$$

$$(1/l_1) + (1/l_2) = 1/f$$

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Additional Definitions

With a focal point:

$$\begin{pmatrix} x_4 \\ \tan(\alpha_4) \end{pmatrix} = \begin{pmatrix} 1-(l_2/f) & 0 \\ -1/f & 1-(l_1/f) \end{pmatrix} \begin{pmatrix} x_1 \\ \tan(\alpha_1) \end{pmatrix}$$

With:
$1/f = (1/l_1) + (1/l_2)$

$$\frac{x_4}{x_1} = M = 1 - \frac{l_2}{f} = (-\,l_2/l_1)$$

M is called the magnification

Notation change to allow for generalization

$$\begin{pmatrix} x(z) \\ \tan(\alpha(z)) \end{pmatrix} = \begin{pmatrix} (x_2|x_1) & (x_2|\tan(\alpha_1)) \\ (\tan(\alpha_1)|x_1)) & (\tan(\alpha_2)|\tan(\alpha_1)) \end{pmatrix} \begin{pmatrix} x_1 \\ \tan(\alpha_1) \end{pmatrix}$$

$$\begin{pmatrix} x(z) \\ a(z) \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) \\ (a|x)) & (a|a) \end{pmatrix} \begin{pmatrix} x_1 \\ a_1 \end{pmatrix}$$

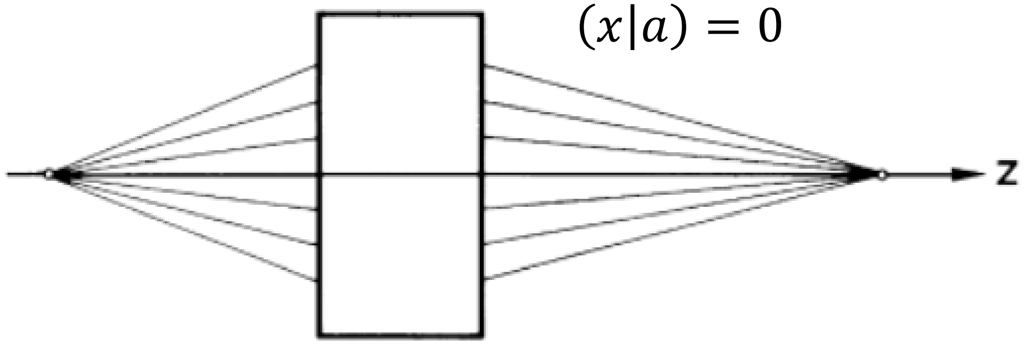
$$a = v_x/c \simeq \tan(\alpha)$$
$$b = v_y/c \simeq \tan(\beta)$$

Can be a simple drift or a very complex system.

Examples

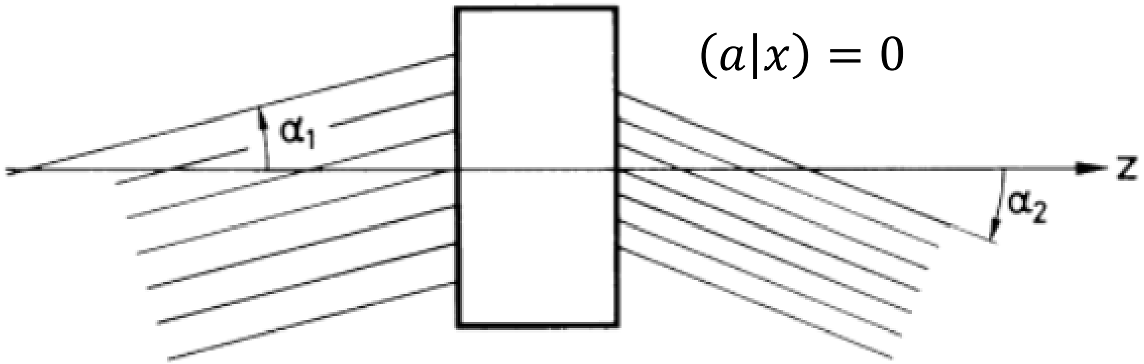
$$\begin{pmatrix} x(z) \\ a(z) \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) \\ (a|x) & (a|a) \end{pmatrix} \begin{pmatrix} x_1 \\ a_1 \end{pmatrix}$$

$(x|a) = 0$



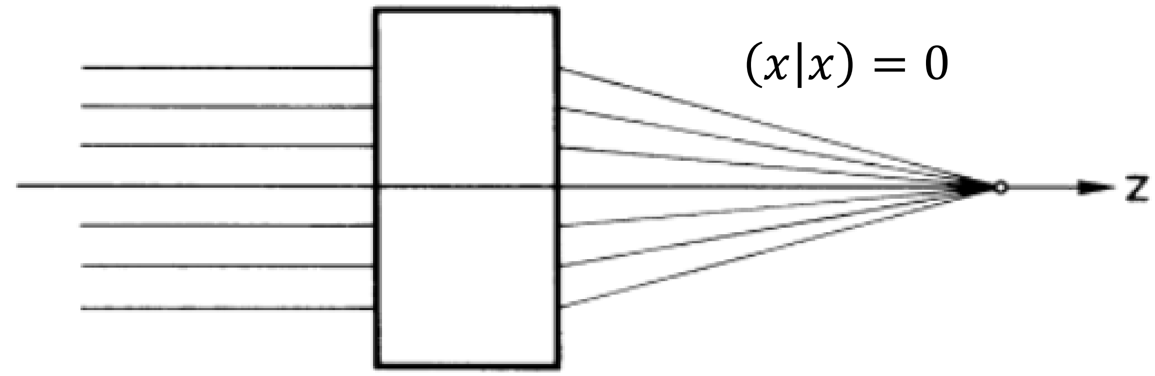
(a)

$(a|x) = 0$



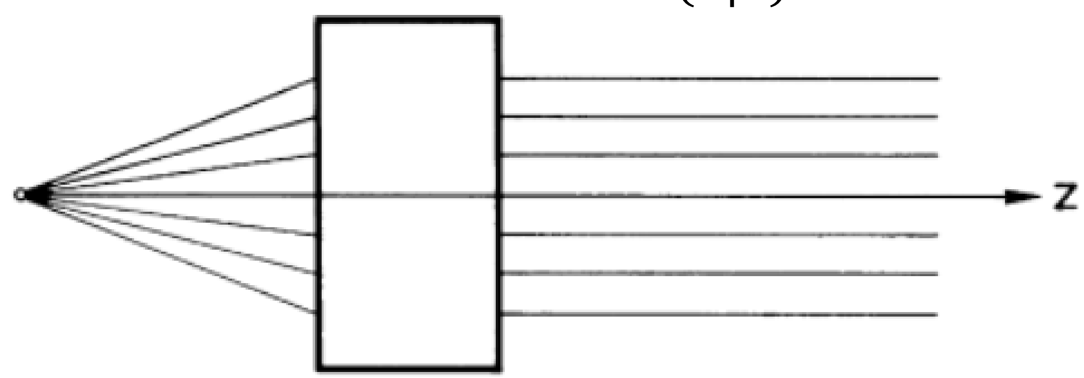
(b)

$(x|x) = 0$



(c)

$(a|a) = 0$



(d)

What constraints on which matrix element needs to explain the plots?

Charged Particle Optics

- Same idea
- $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- In a uniform magnetic field charged particles have a circular motion
  - $mv^2/R = qvB$
  - $B\rho = p/q$ Magnetic Rigidity
- Equivalent for Electric field

Approximation ($\vec{E} \perp$ to trajectory)

  - $E\rho = 2K/q$ Electric Rigidity

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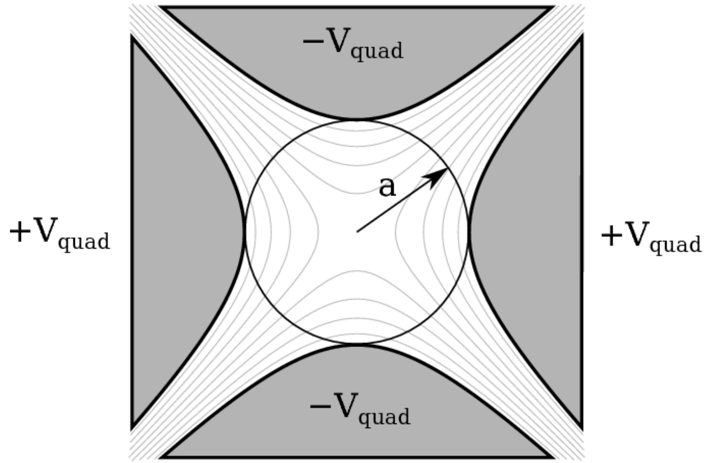
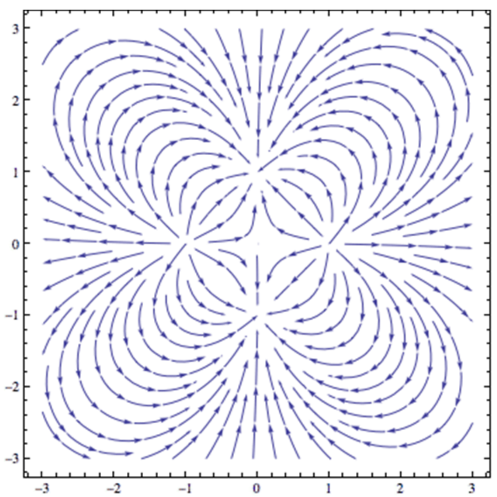
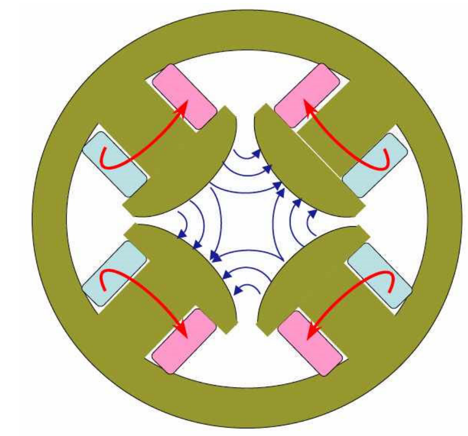
Quadrupole Lenses

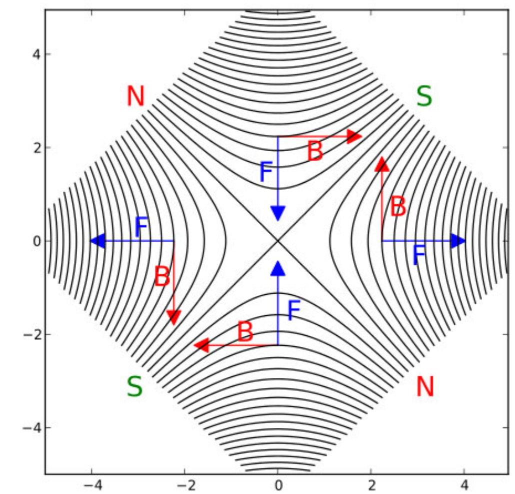


Electric





Magnetic



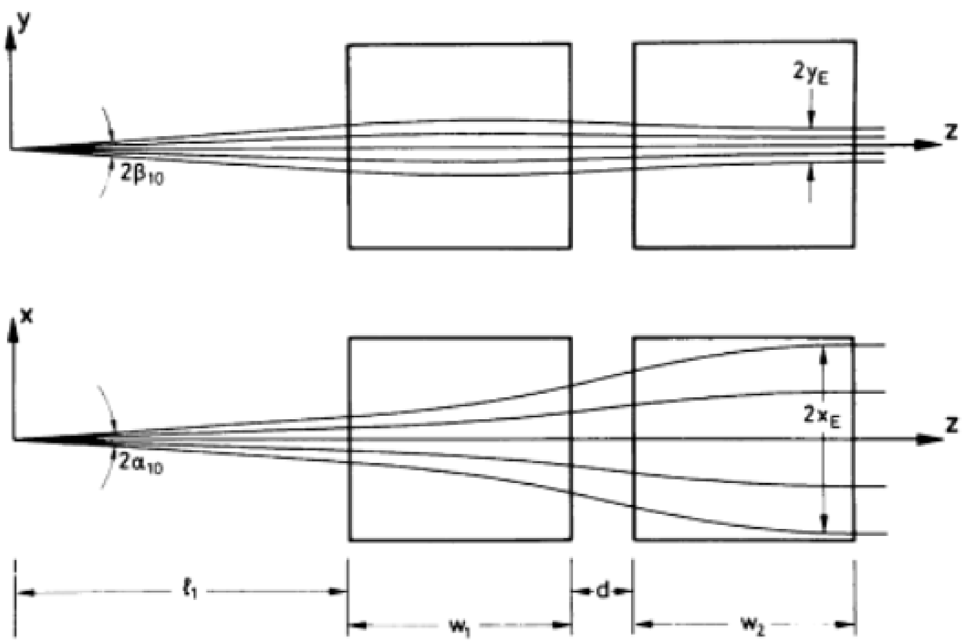
Focus <u>only</u> in one direction

The motion of charged particles in any of the element presented here is known.

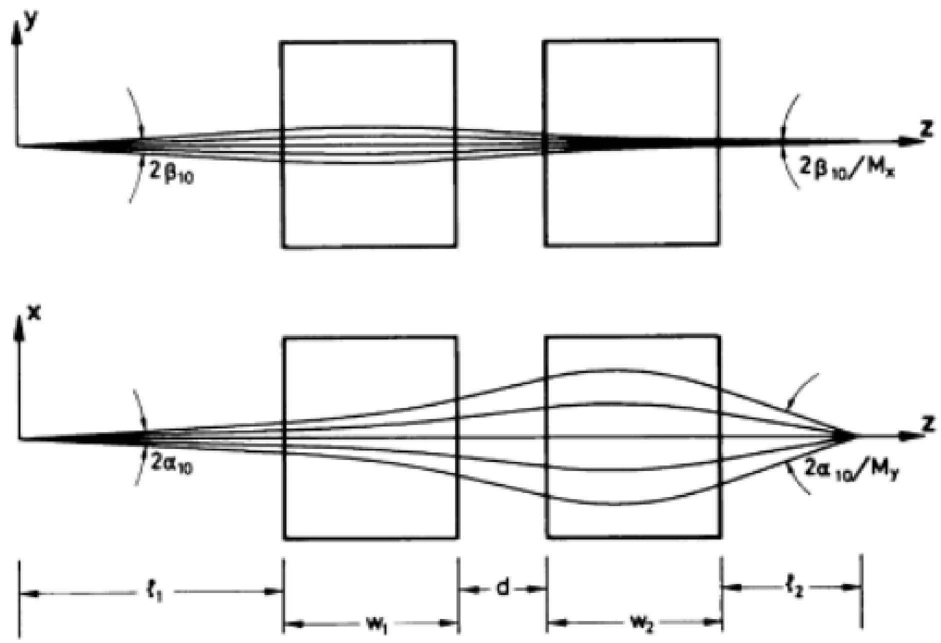
The transformation matrix is known as well.

You can solve the equation of motion to find them.

Combination of Quadrupole Lenses



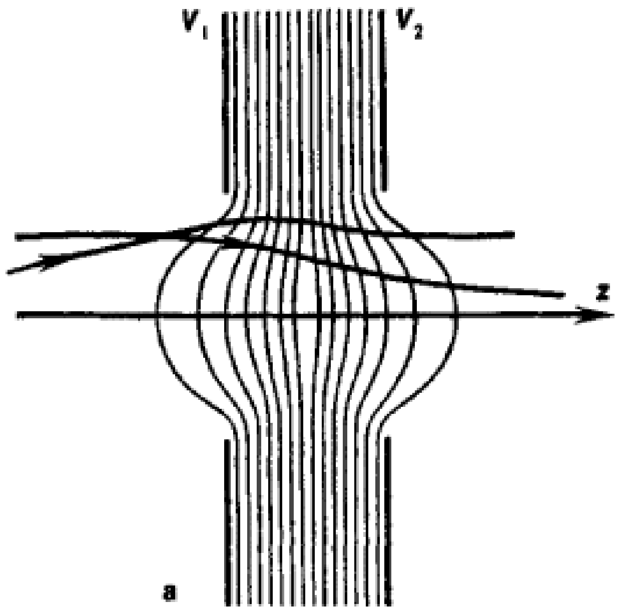
Point to parallel

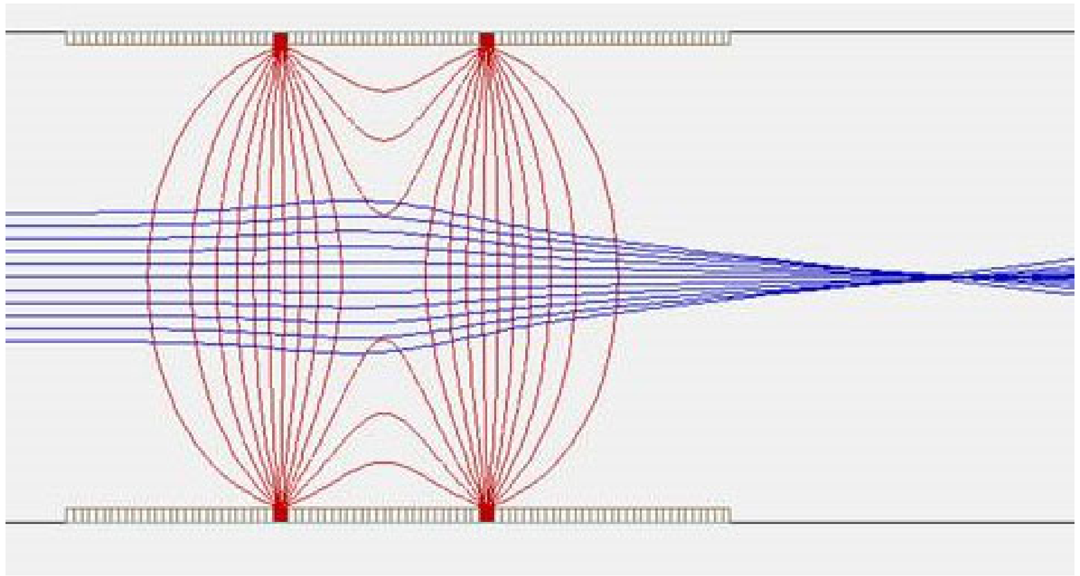


Point to point

With appropriate diagnostic tune by slowly changing one field and correcting with the other

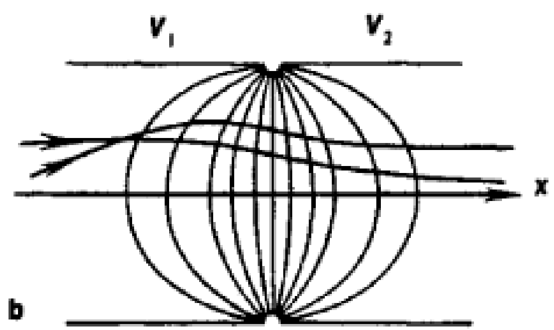
Symmetric Focusing

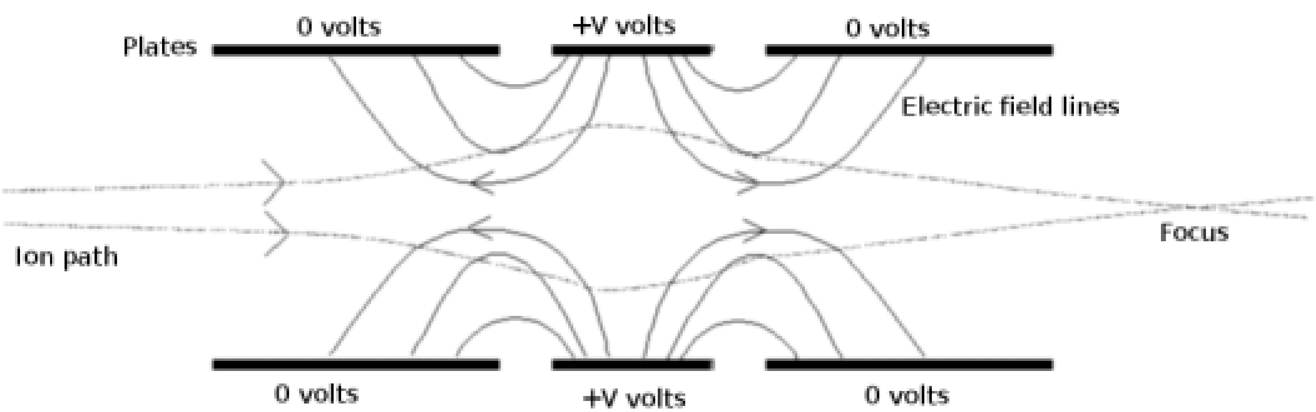




A Single tuning parameter. Maximize transmission to appropriate diagnostic.

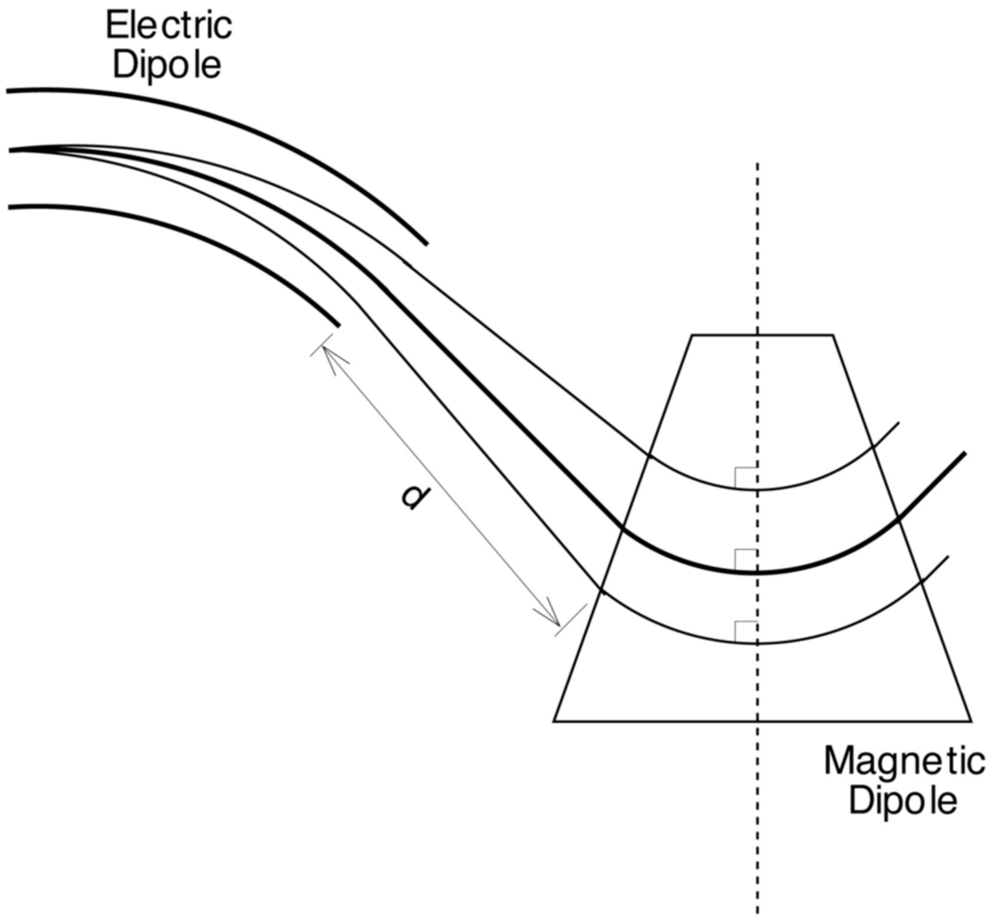
Works at relatively low energies ~100 keV Max

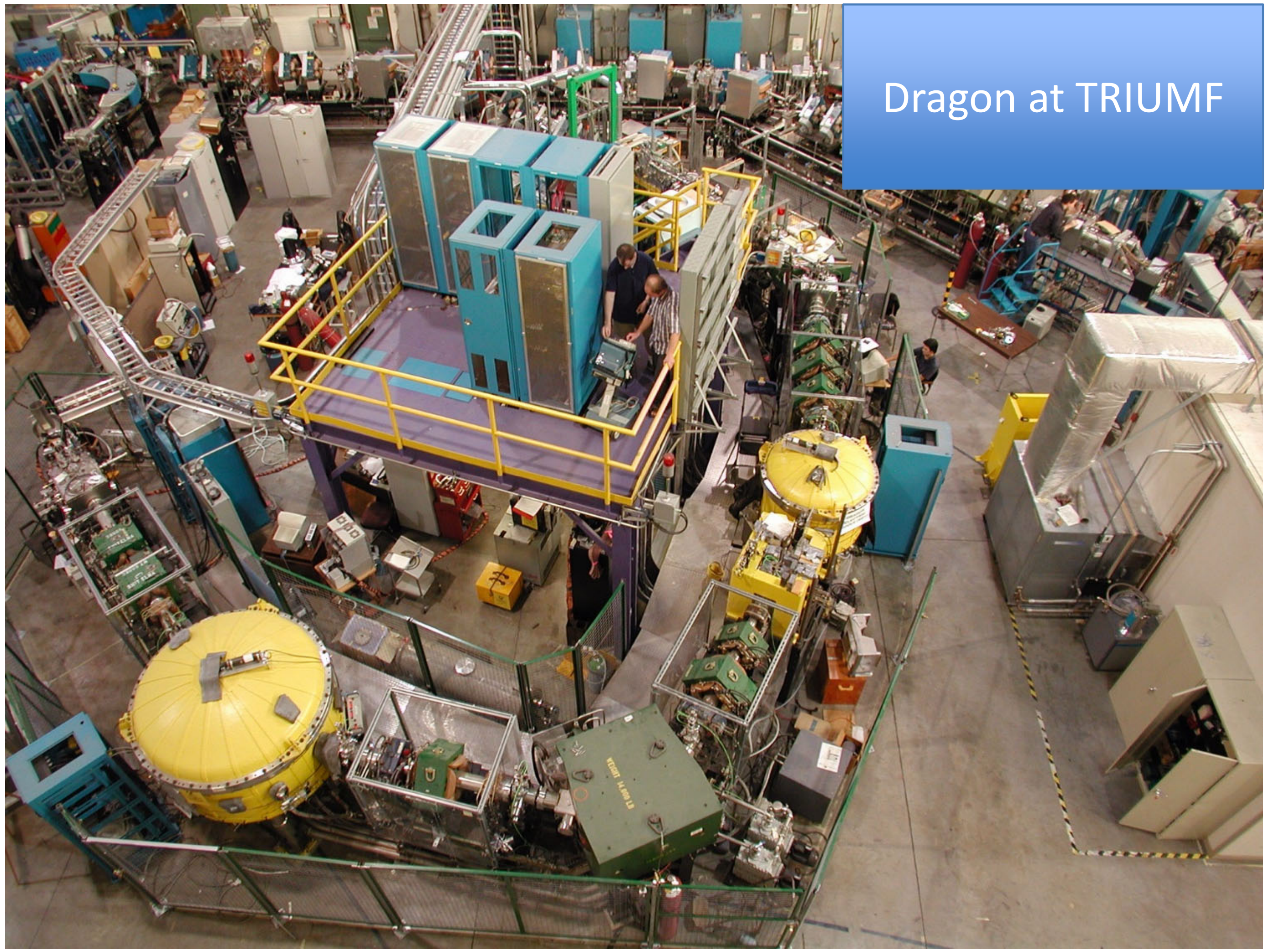




WHY?

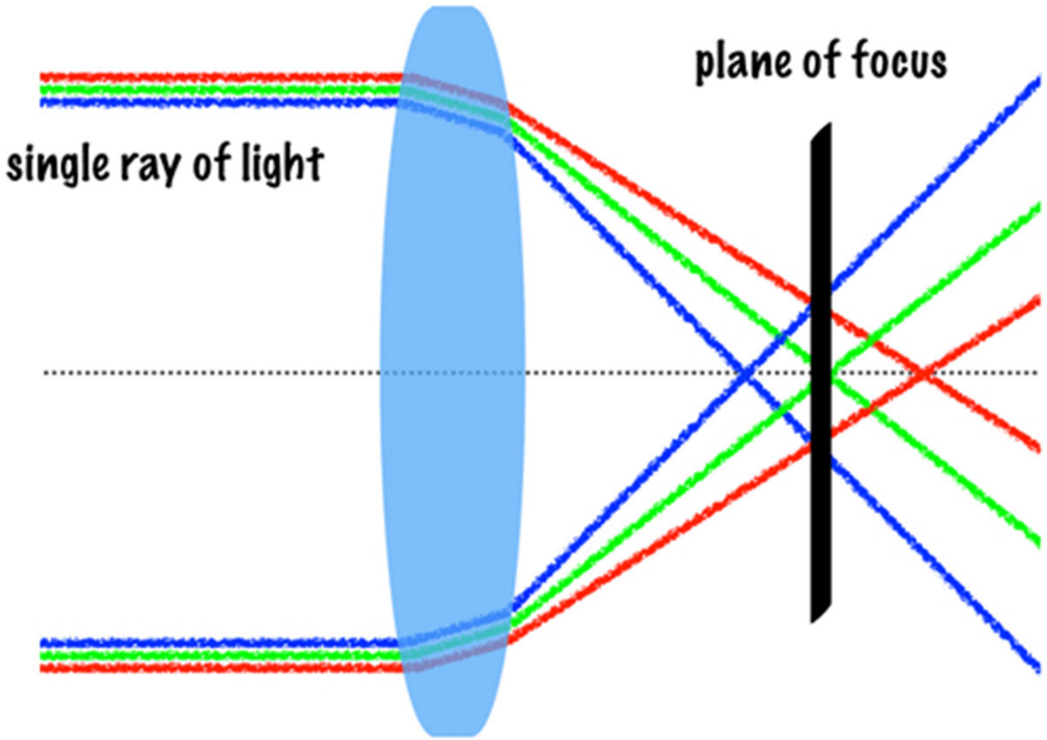
Bending Charged Particles





Dragon at TRIUMF

The Color Twist



d could represent any variable
They could be more
Actually, most transport code
Use <u>at least</u> 6 dimensions.

Most of the matrix element
are zero

$$\begin{pmatrix} x(z) \\ a(z) \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) \\ (a|x)) & (\mathrm{a}|a) \end{pmatrix} \begin{pmatrix} x_1 \\ a_1 \end{pmatrix}$$

$$\begin{pmatrix} x(z) \\ a(z) \\ d(z) \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) & (x|d) \\ (a|x) & (a|a) & (a|d) \\ (d|x) & (d|a) & (d|d) \end{pmatrix} \begin{pmatrix} x_1 \\ a_1 \\ d_1 \end{pmatrix}$$

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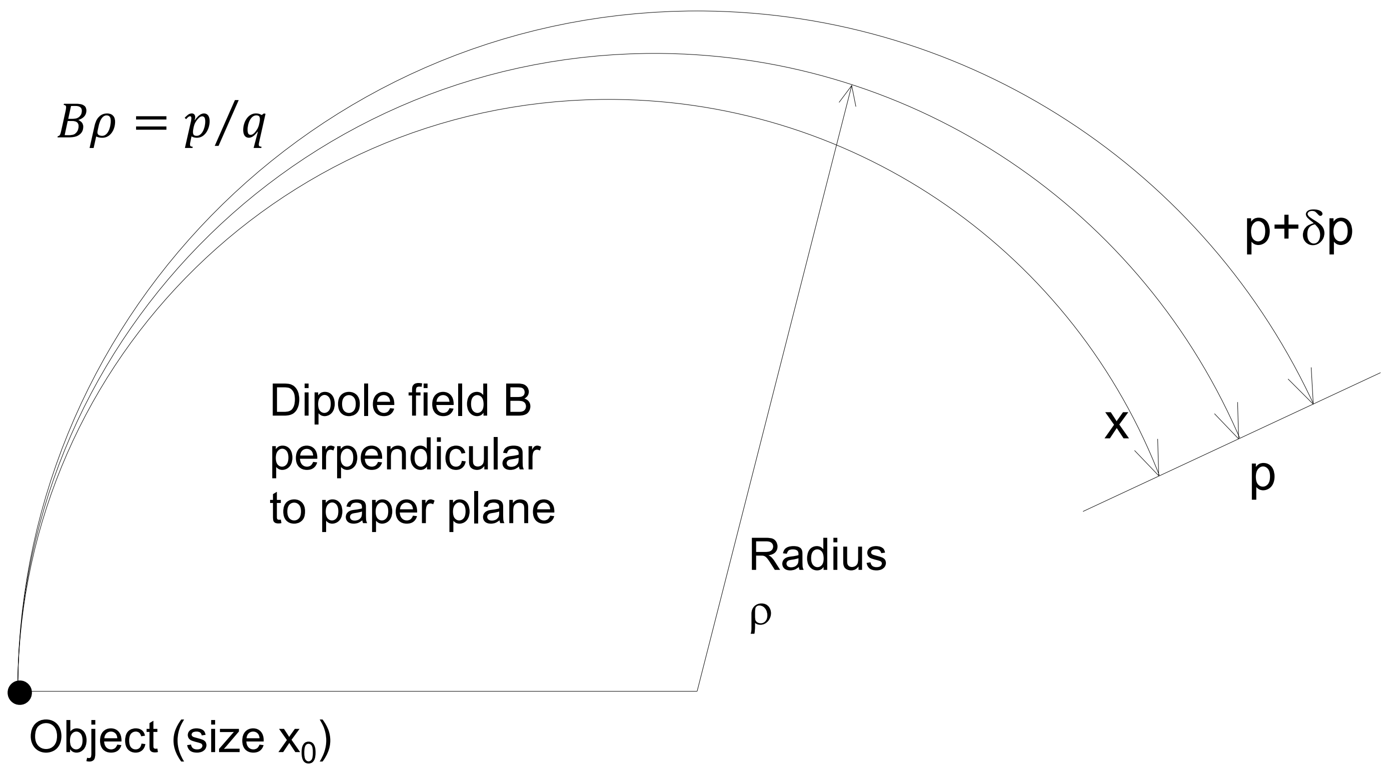
ISNAP

Dispersion



Here $d = {}^{\delta p}/_{p_0}$

Spatial Dispersion δx/δp
used in **magnetic analysis**

Other/additional potential dispersion:

- Mass
- Energy
- Charge

Resolving Power

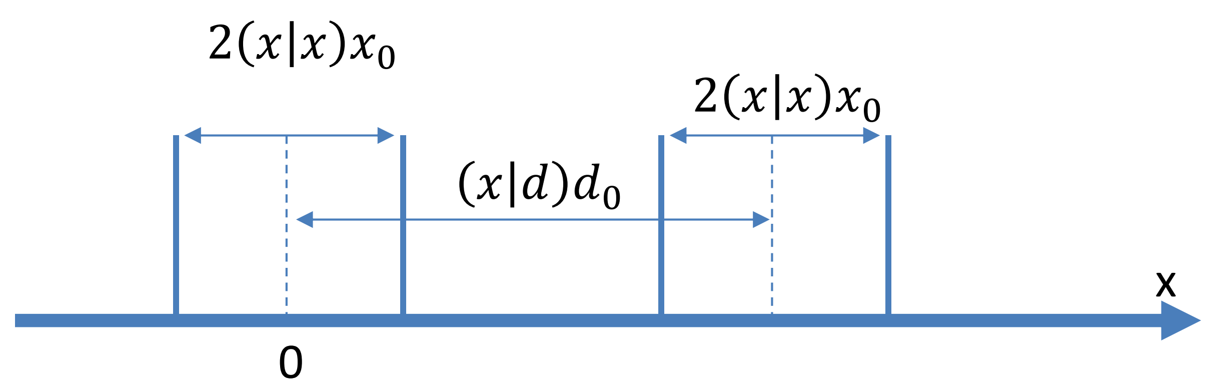
$$\begin{pmatrix} x(z) \\ a(z) \\ d(z) \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) & (x|d) \\ (a|x) & (a|a) & (a|d) \\ (d|x) & (d|a) & (d|d) \end{pmatrix} \begin{pmatrix} x_0 \\ a_0 \\ d_0 \end{pmatrix}$$

d : $\Delta p/p_0$, $\Delta m/m_0$, $\Delta E/E_0$ or $\Delta Q/Q_0$

Your favorite spectrometer/separator

$$x(z) = (x|x)x_0 + \cancel{(x|a)}a_0 + (x|d)d_0$$

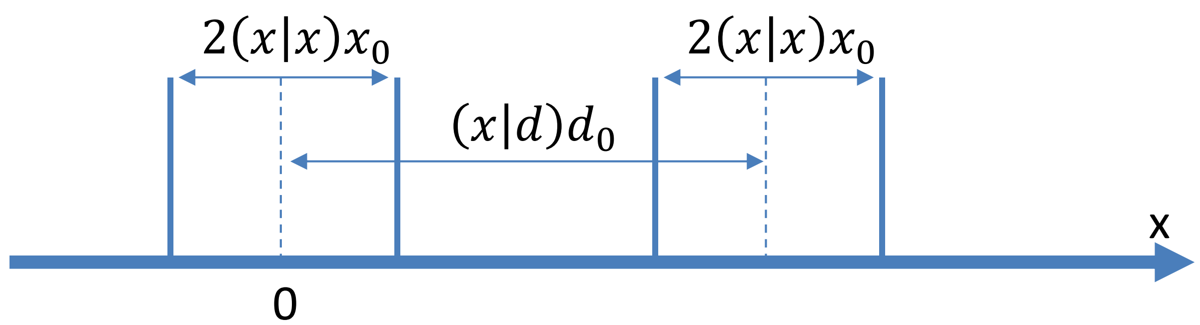
Maximum half size of your target



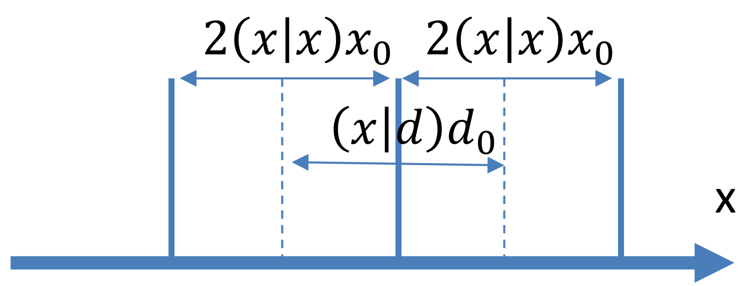
Resolving Power

$$x(z) = (x|x)x_0 + \cancel{(x|a)}a_0 + (x|d)d_0$$

Half size of your target



The resolving power is the inverse value of d_0 that still provide a separation



$$(x|d)d_0 = 2(x|x)x_0$$

$$1/d_0 = \frac{(x|d)}{2(x|x)x_0}$$

MDM @ Texas A&M

Table 1
Major parameters of the Oxford Spectrometer

Maximum mass–energy product (ME/q^2)	315 MeV amu
Vertical acceptance	±50 mrad
Horizontal acceptance	±40 mrad
Solid angle (max.)	8 msr
Energy bite E_{max}/E_{min} ($\Omega = 1$ msr)	1.52
Energy bite E_{max}/E_{min} ($\Omega = 8$ msr)	1.31
Length of focal plane	0.69 m
Angle of focal plane	normal to incident ions
Dispersion	
$k = 0.0$ [a]	3.8 cm/%
$k = 0.3$	3.3 cm/%
Horizontal angular magnification ($k = 0$)	−2.5
Horizontal linear magnification ($k = 0$)	−0.4
Vertical linear magnification ($k = 0$)	5.0
First order resolving power ($E/\mathrm{d}E$) (calculated)	4500
Range of kinematic compensation	
min.	$k = -0.1$
max	$k = 0.3$
Dipole field gradient, α [b]	−0.191
Maximum dipole field level	1.35 T
Angle of deflection	100°
Central radius, R	1.6 m

[a] k is defined as $-(1/p)(\mathrm{d}p/\mathrm{d}\theta)$, the fractional kinematic variation of momentum with scattering angle. Note that the sign of k changes on either side of the beam direction.

[b] As is conventional, the gradient field parameterisation is $B(r) = B_0\,[1 + \alpha(x/R_0) + \beta(x/R_0)^2 + \cdots]$ where $x = r - R_0$ and B_0 is the field at the central radius R_0.

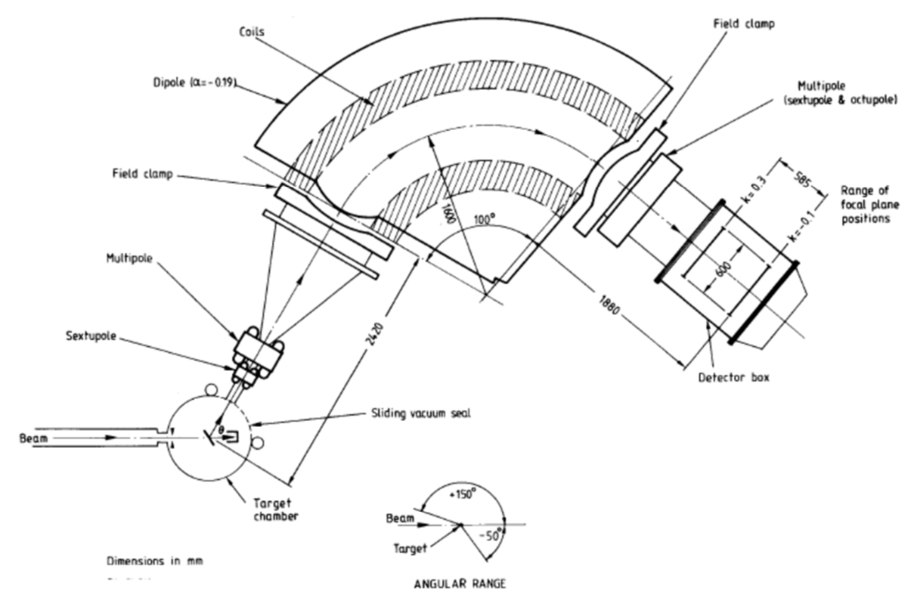
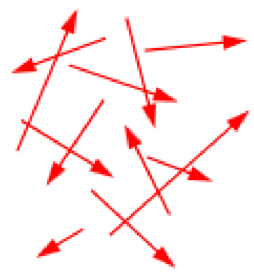
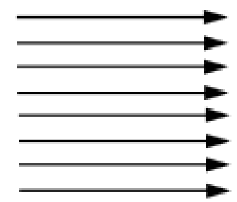


Fig. 1. The layout of the MDM-2 spectrometer system showing all of the magnetic elements.

Beam?

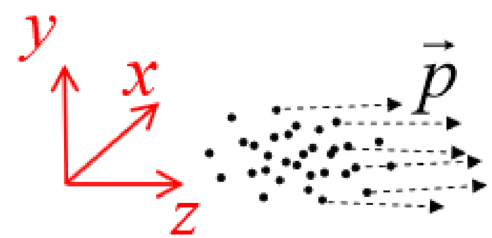


plasma

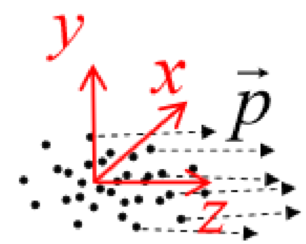


beam

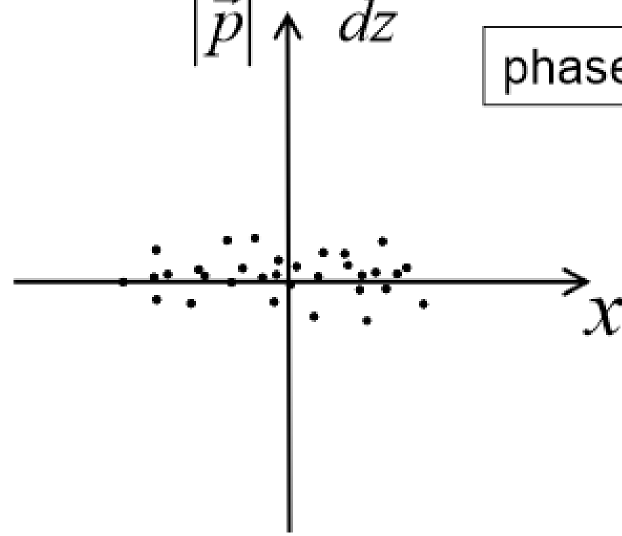
beam



Beam?



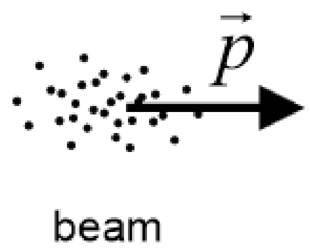
$$\frac{p_x}{|\vec{p}|} = \frac{dx}{dz} \cong x'$$

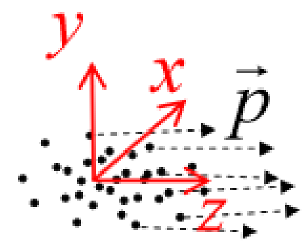


phase space diagram

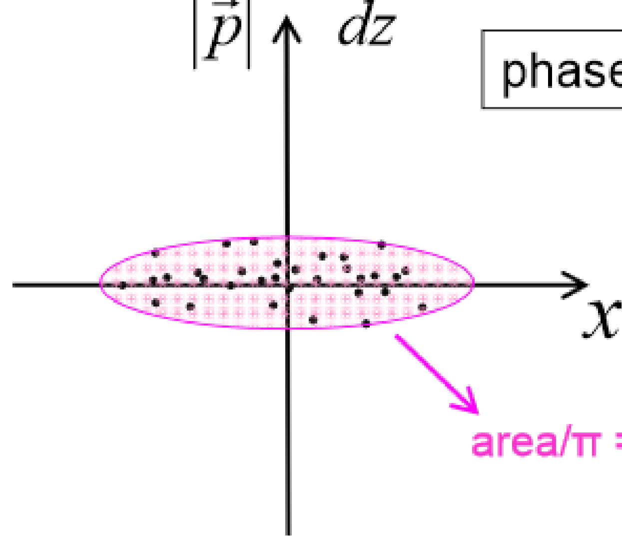
Same for the vertical plane

Concept of emittance





$$\frac{p_x}{|\vec{p}|} = \frac{dx}{dz} \cong x'$$



phase space diagram

emittance definition:

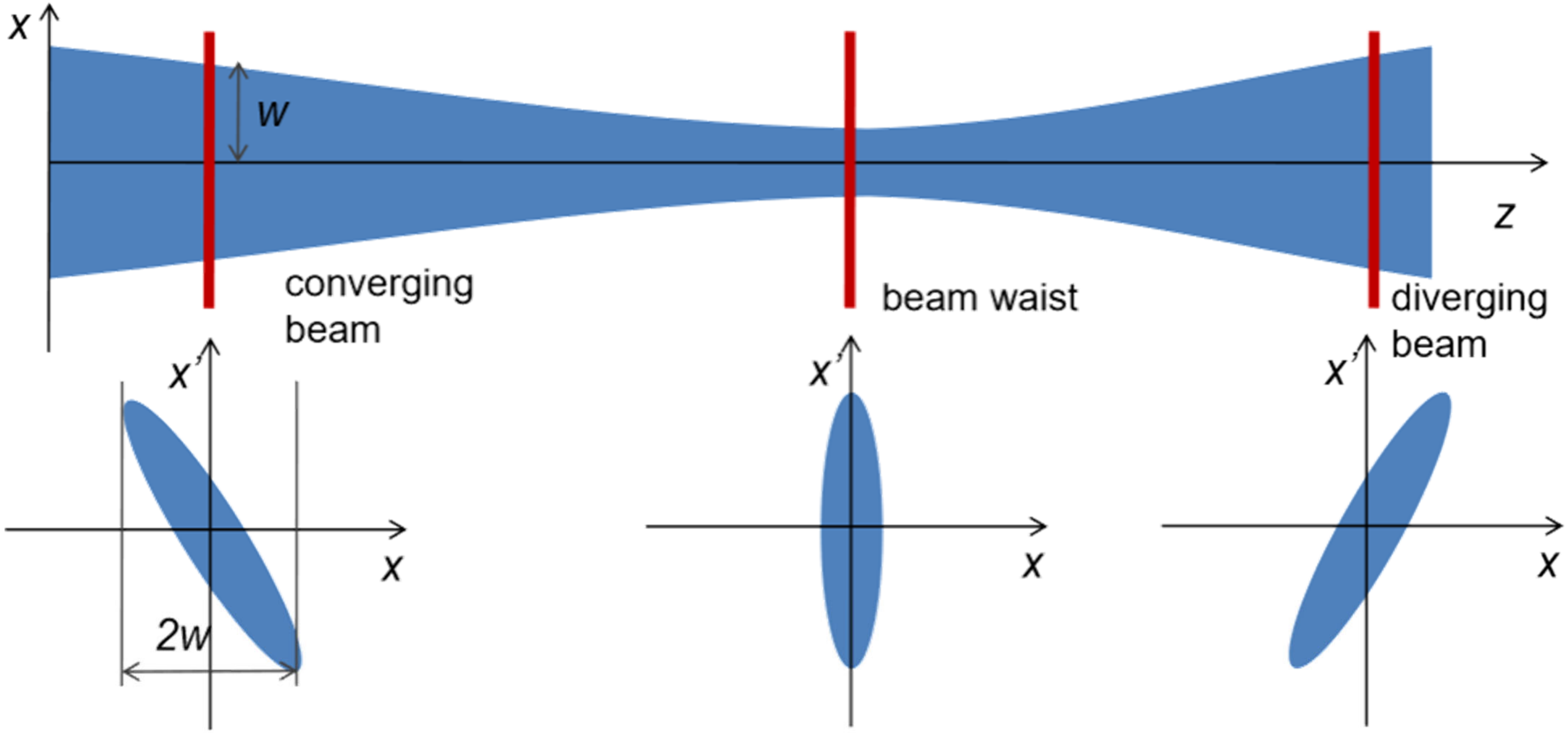
$$\varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

area/π = emittance (units: mm.mrad)

The emittance is conserved as long as the forces to which the beam is subjected to are conservative.

Same for the vertical plane

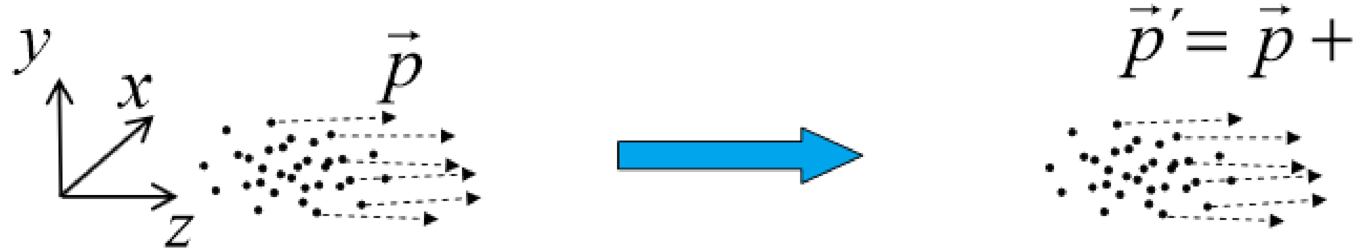
Concept of emittance



Along a beamline the orientation and aspect ratio of beam ellipse in x, x' plane varies, but area $\pi\varepsilon$ remains constant

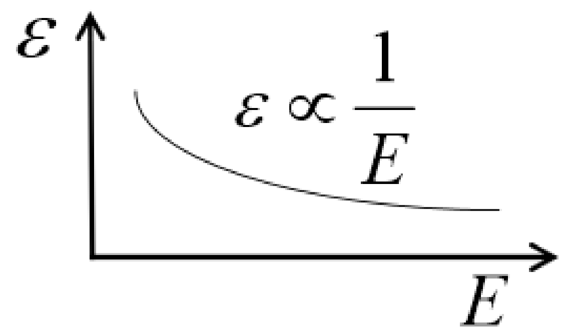
Emittance

$$\vec{p}' = \vec{p} + \Delta\vec{p}_z$$



acceleration (only in z direction) $\frac{p_x}{|\vec{p}|}$ gets smaller

linear accelerators:



definition:

normalized emittance:

$\epsilon_N = \epsilon . \beta . \gamma$ =constant

geometrical emittance

The normalized emittance is conserved during acceleration

Emittance and Radioactive Beam Production Method

- Depending on the production method
- Emittance of RI Beam can be much worse than the one of stable beam
  - Equipment have to be designed to accommodate
    - Large energy and angular acceptance
    - Larger bore
- ReA3(6,12), Caribu, Twinsol …

Equivalence of Transport of One Ray ⇔ Ellipse

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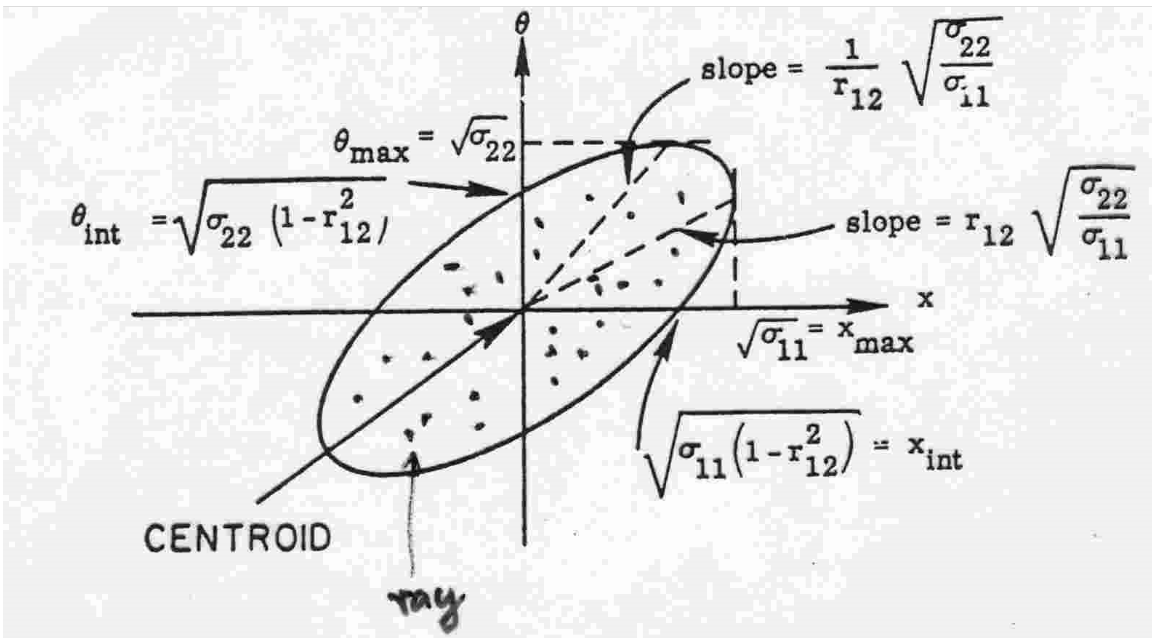
Defining the σ Matrix representing a Beam

The 2-dimensional case (x, Θ)=(x,x')



$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

$$r_{21} = r_{12} = \frac{\sigma_{21}}{\sqrt{\sigma_{11}\,\sigma_{22}}}$$

Real, pos. definite symmetric **σ Matrix**

$$\epsilon = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} = \det(\sigma)^{1/2}$$

$$\sigma^{-1} = 1/\varepsilon^2 \begin{pmatrix} \sigma_{22} & -\sigma_{21} \\ -\sigma_{21} & \sigma_{11} \end{pmatrix}$$

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Equivalence of Transport of One Ray ⇔ Ellipse

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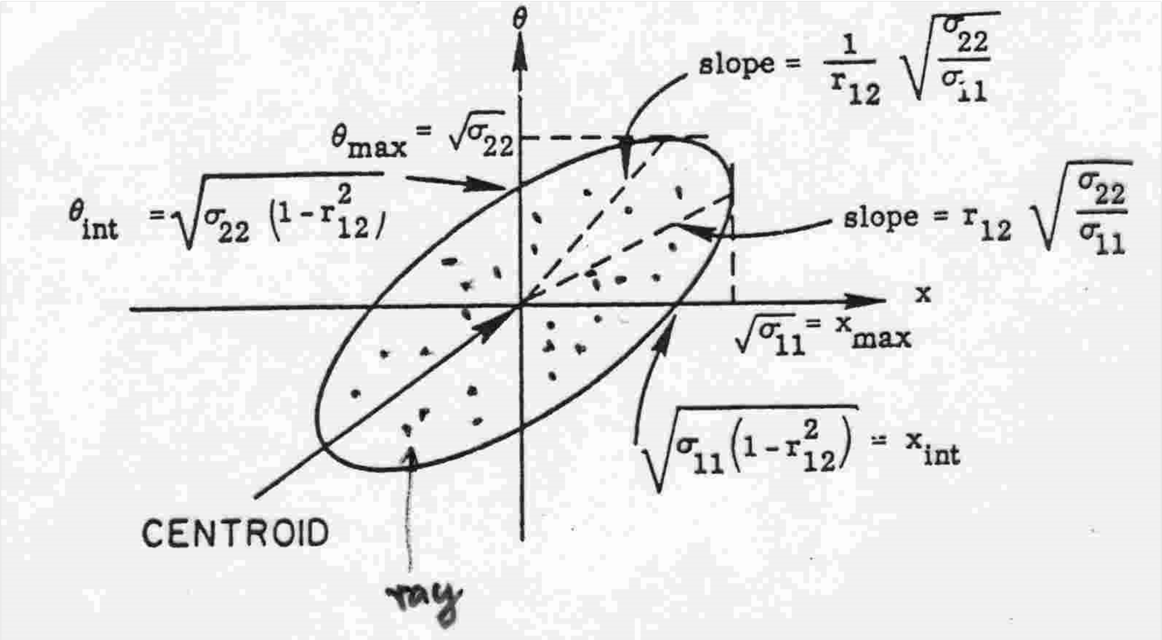

2-dim. Coord.vectors
(point in phase space)

$$X = \begin{pmatrix} x \\ \Theta \end{pmatrix}$$

$$X^T = (x\ \Theta)$$

Ellipse in Matrix notation:

$$X^T \sigma^{-1} X = 1$$

Exercise: Show that Matrix notation is equivalent to known Ellipse equation:

$$\sigma_{22}\, x^2 - 2\sigma_{21}\, x\,\Theta + \sigma_{11}\Theta^2 = \varepsilon^2$$

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Beam Sigma Matrix and Transfer Matrix

Ray X_0 from location 0 is transported by a 6 x 6 Matrix R to location 1 by: $X_1 = RX_0$ **(1)**

Note: R maybe a matrix representing a complex system is : $R = R_n R_{n-1} \ldots R_0$

Ellipsoid in Matrix notation, generalized to e.g. 6-dim. using σ Matrix: $X_0^{\,T} \sigma_0^{-1} X_0 = 1$ **(2)**

Inserting Unity Matrix $I = RR^{-1}$ in equ. **(2)** it follows $X_0^{\,T} (R^T R^{T-1}) \sigma_0^{-1} (R^{-1} R) X_0 = 1$

from which we derive $(RX_0)^T (R\sigma_0 R^T)^{-1} (RX_0) = 1$

The equation of the new ellipsoid after transformation becomes $X_1^{\,T} \sigma_1^{-1} X_1 = 1$

where $\sigma_1 = R\sigma_0 R^T$ **(3)**

Conclusion: Knowing the TRANSFER matrix R that transports one ray through an ion-optical system using **(1)** we can now also transport the phase space ellipse describing the initial beam using **(3)**

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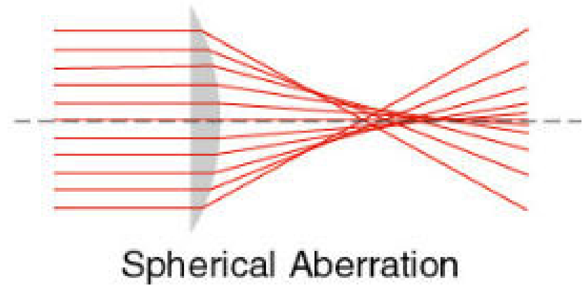
Beam/Ion Transport Calculation

- Now we have a "framework" to calculate either individual particle trajectories or a full beam…
  – How do we use it? Do we have to calculate individual transfer matrix?
  – Code with various approach and level of details
    - TRANSPORT
    - COSY INFINITY
    - GIOS
    - GICOSY
    - …
  – SIMION, OPERA, …

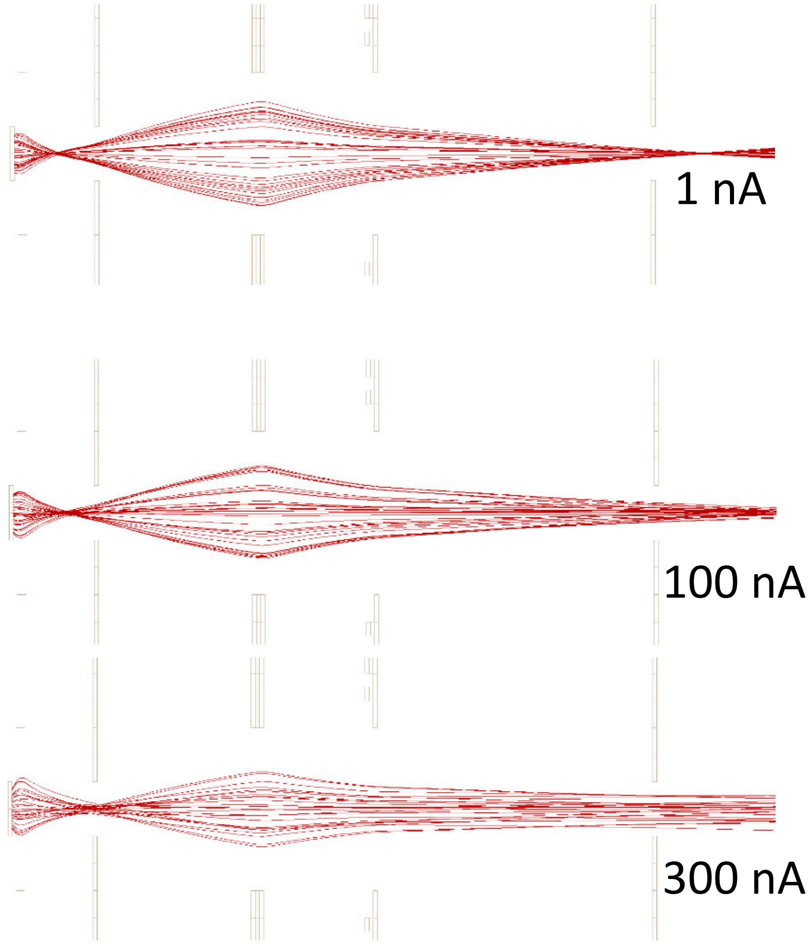
What Did we Ignore

Truncation of equation of motion to allow for linear combination and usage of matrix formalism. Truncation is similar to selection of order of a Taylor expansion. Missing parts are called “Higher Order”

Element “limitation”



What Did we Ignore



Beam Space Charge Effects
Space charge in the accelerating lens can cause broadening of the beam, which will affect all of the ions in the beam, independent of mass.

What to Remember

- Want to get best beam ever
  - Speak with your operators and speak their vocabulary
  - Ask them questions
- Your experiments depends on beams, spectrometers, separators
  - The ability to calculate trajectories and tune is a big advantage
  - Talk to people to get going with such calculations
    - Large group at MSU/NSCL/FRIB and ANL/ATLAS
    - Smaller group scattered around the country
    - Dedicated accelerator groups at all the national labs